## Last Name:

First name:

## Student ID:

1. Solve each of the following equations.
(a) $x^{2} y \frac{d y}{d x}=e^{\frac{1}{x}} \sec (y)$
[8]
(b) $t^{2} \frac{d y}{d t}-2 t^{2} \tan (t) y=5 \sec ^{2}(t), \quad y(\pi)=0$
[6]
(c) $\frac{d^{4} y}{d x^{4}}-5 \frac{d^{2} y}{d x^{2}}-36 y=0$
[4] 2. Determine an integrating factor which will make the equation

$$
x y+y^{2}+(x+2 y-1) \frac{d y}{d x}=0
$$

exact.
[8] 3. Use the method of undetermined coefficients to find the general solution of

$$
\frac{d^{2} y}{d x^{2}}-3 \frac{d y}{d x}-4 y=15 e^{4 x}+16 x
$$

[8] 4. Use the method of variation of parameters to find the general solution of

$$
x^{2} \frac{d^{2} y}{d x^{2}}+2 x \frac{d y}{d x}-6 y=\ln (x),
$$

given that $y_{1}=x^{2}$ and $y_{2}=x^{-3}$ are solutions of the homogeneous equation

$$
x^{2} \frac{d^{2} y}{d x^{2}}+2 x \frac{d y}{d x}-6 y=0 .
$$

[8] 5. Find $\lambda$ so that equation $y^{\prime \prime}+\lambda y=0, y(0)=y(1)=0$ has non-zero solution.
[6] 6. For a sample space $\Omega=\{1,2,3,4,5,6\}$. Find two $\sigma$-algebra $\mathbb{F}$. What is the $\sigma$-algebra $\mathbb{F}$ that contains the maximum numbers of subsets.
[8] 7. (1). Find the integral $\int_{-\infty}^{\infty} e^{-x^{2}} d x$.
(2). Find the expectation and variance of the Normal distribution.

