MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

Assignment 5 Solutions **Mathematics 2050** **fall 2018**

1. $A = \begin{bmatrix} 3-\cos\frac{\pi}{6} & 6-\cos\frac{2\pi}{6} \\ 6-\cos\frac{\pi}{6} & 12-\cos\frac{2\pi}{6} \\ 9-\cos\frac{\pi}{6} & 18-\cos\frac{2\pi}{6} \end{bmatrix} = \begin{bmatrix} 3-\frac{\sqrt{3}}{2} & 6-\frac{1}{2} \\ 6-\frac{\sqrt{3}}{2} & 12-\frac{1}{2} \\ 9-\frac{\sqrt{3}}{2} & 18-\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 3-\frac{\sqrt{3}}{2} & \frac{11}{2} \\ 6-\frac{\sqrt{3}}{2} & \frac{23}{2} \\ 9-\frac{\sqrt{3}}{2} & \frac{35}{2} \end{bmatrix}$

2. $A = \begin{bmatrix} \mathbf{v} & \mathbf{w} \\ \downarrow & \downarrow \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 4 \\ 3 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} \mathbf{v}^T & \rightarrow \\ \mathbf{w}^T & \rightarrow \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & -5 \end{bmatrix}$. Therefore,

   $a_{11} = 1$, a_{13}(not possible), $a_{21} = 2$ and
   b_{32}(not possible), $b_{12} = 2$, $b_{22} = 4$

3. We seek a, b and c such that $\begin{bmatrix} -1 \\ 14 \\ 2 \end{bmatrix} = a \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \text{b} \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} + \text{c} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. Hence we have the following system of equations:

$$\begin{aligned} a + 2b \quad &= -1 \\ 3b + c &= 14 \\ c &= 2 \end{aligned}$$

   It follows that $3b + 2 = 14 \Rightarrow 3b = 12 \Rightarrow b = 4$ and $a + 8 = -1 \Rightarrow a = -9$.

   hence, $A \begin{bmatrix} -9 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 14 \\ 2 \end{bmatrix}$

4. $\left(3A^T + 3\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\right)^T = \begin{bmatrix} 8 & 0 \\ 3 & 1 \end{bmatrix}$

   $(3A^T)^T + \begin{bmatrix} 3 & 0 \\ 0 & 6 \end{bmatrix}^T = \begin{bmatrix} 8 & 0 \\ 3 & 1 \end{bmatrix}$

   $3A + \begin{bmatrix} 3 & 0 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 3 & 1 \end{bmatrix}$

   $3A = \begin{bmatrix} 5 & 0 \\ 3 & -5 \end{bmatrix} \Rightarrow A = \frac{1}{3}\begin{bmatrix} 5 & 0 \\ 3 & -5 \end{bmatrix}$.

5. (a) $\begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 0 & -2 \end{bmatrix}$

(b) $\begin{bmatrix} 5 & 0 & -7 \\ 1 & 5 & 9 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 1 & 9 & 7 \\ -1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 17 & 15 & -9 \\ -2 & 48 & 54 \end{bmatrix}$.

6. $AB = BA \iff \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix} \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix} \iff \begin{bmatrix} 23 & 5k-10 \\ -9 & k+15 \end{bmatrix} =$ $\begin{bmatrix} 23 & 15 \\ -3k+6 & k+15 \end{bmatrix} \iff 5k - 10 = 15$ and $-3k + 6 = -9$. Therefore, $k = 5$.

7. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be any 2×2 matrix.

   A commutes with $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \iff \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \iff \begin{bmatrix} b & 0 \\ d & 0 \end{bmatrix}$ $= \begin{bmatrix} 0 & 0 \\ a & b \end{bmatrix} \iff b = 0$ and $a = d$.

8. $4Q = I - 2P$
   $Q = \frac{1}{4}(I - 2P)$. Hence,

$$\begin{aligned} Q^2 &= \tfrac{1}{16}(I - 2p)^2 \\ &= \tfrac{1}{16}(I - 2P)(I - 2P) \\ &= \tfrac{1}{16}(I^2 - 2PI - 2IP + 4P^2) \\ &= \tfrac{1}{16}(I - 2P - 2P + 4P) \\ &= \tfrac{1}{16}I \end{aligned}$$