MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Assignment 1 **Mathematics 2050** **Fall 2018**

Due: Sept 21, 2018 SHOW ALL WORK

[6] 1. Vector $\vec{w}$ has the starting point $(1, 2, 3)$ and the ending point $(-4, 7, 5)$. which of the following vectors is parallel to $\vec{w}$?

(a) $\vec{u} = \begin{bmatrix} 3 \\ 4 \\ -1 \end{bmatrix}$, (b) $\vec{v} = \begin{bmatrix} 10 \\ -10 \\ -4 \end{bmatrix}$ (c) $\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Solution: $\vec{w} = \begin{bmatrix} -4-1 \\ 7-2 \\ 5-3 \end{bmatrix} = \begin{bmatrix} -5 \\ 5 \\ 2 \end{bmatrix}$. $\vec{v} = 2\vec{w}$. Therefore, $\vec{v}$ is parallel to $\vec{w}$.

$\vec{x} = 0\vec{w}$. $\vec{x}$ is parallel to $\vec{w}$.

$\vec{u}$ cannot be expressed as a scalar multiple of $\vec{w}$. $\vec{u}$ is not parallel to $\vec{w}$.

[4] 2. The shape $ABCD$ is a parallelogram where $A(1, 0, 0), B(2, 1, 1),\ C(3, 4, 5)$. Find the coordinate of the point D. What is the coordinate of the intersection point of AC and BD?

Solution: $\vec{AB} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\vec{BC} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$. By $\vec{AD} = \vec{BC}$, we can get $D(2, 3, 4)$.

The intersection point is the mid-point of BD. Its coordinate is $(2, 2, \frac{5}{2})$.

[4] 3. Let $P_1(1, 2, -1)$ and $P_2(3, 4, 5)$. Find the point of P on the line segment of P_1 and P_2, so that P_2P is $\frac{1}{4}$ of P_2P_1.

Solution: Set $P(x, y, z)$. Then $\vec{P_2P_1} = \begin{bmatrix} 1-3 \\ 2-4 \\ -1-5 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ -6 \end{bmatrix}$, $\vec{P_2P} = \begin{bmatrix} x-3 \\ y-4 \\ z-5 \end{bmatrix}$. By $\vec{P_2P} = \frac{1}{4}\vec{P_2P_1}$, we get

$$\begin{aligned} x-3 &= \tfrac{1}{4}(-2) \\ y-4 &= \tfrac{1}{4}(-2) \\ z-5 &= \tfrac{1}{4}(-6) \end{aligned}$$

and $(x, y, z) = (\frac{5}{2}, \frac{7}{2}, \frac{7}{2})$.

[6] 4. (a) Is $\vec{w} = \begin{bmatrix} 1 \\ 10 \end{bmatrix}$ a linear combination of $\vec{u} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$?

(b) Can every vector $\vec{w} = \begin{bmatrix} a \\ b \end{bmatrix}$ be written as a linear combination of $\vec{u}$ and $\vec{v}$?

Solution: (a) Let $\vec{w} = k_1\vec{u} + k_2\vec{v}$. This gives

$$\begin{aligned} 1 &= k_1 + 2k_2 \\ 10 &= 4k_1 + 3k_2 \end{aligned}.$$

Solving this system, we obtain

$$k_1 = \frac{17}{5},\ k_2 = \frac{-6}{5}.$$

Therefore $\vec{w}$ is a linear combination of $\vec{u}$ and $\vec{v}$.

(b) SimilarlyLet $\vec{w} = k_1\vec{u} + k_2\vec{v}$. We have

$$a = k_1 + 2k_2, \quad b = 4k_1 + 3k_2.$$

Solving this system, we obtain

$$k_2 = \frac{4a-b}{5}, \quad k_1 = a - 2k_2 = a - \frac{2(4a-b)}{5} = \frac{-3a+2b}{5}.$$

This shows that for any a and b, we can find k_1 and k_2 so that $\vec{w}$ is a linear combination of $\vec{u}$ and $\vec{v}$.

[6] 5. In each of the following case, either express p as a linear combination of u, v, w, or explain why there is no such linear combination.

(a) $p = \begin{bmatrix} 4 \\ 5 \\ 3 \end{bmatrix}, \ u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \ v = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \ w = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

(b) $p = \begin{bmatrix} -1 \\ 2 \\ 4 \\ 0 \end{bmatrix}, \ u = \begin{bmatrix} 3 \\ 7 \\ 0 \\ -4 \end{bmatrix}, \ v = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 9 \end{bmatrix}, \ w = \begin{bmatrix} 3 \\ 1 \\ 4 \\ 5 \end{bmatrix}$

Solution: (a) Let $\bar{p} = k_1\vec{u} + k_2\vec{v} + k_3\vec{w}$. We have

$$\begin{aligned} 4 &= k_1 + k_2 + k_3 \\ 5 &= k_2 + k_3 \\ 3 &= k_3. \end{aligned}$$

Therefore we have $k_1 = -1, k_2 = 2, \ k_3 = 3$.

(b) Let $\bar{p} = k_1\vec{u} + k_2\vec{v} + k_3\vec{w}$. We have

$$\begin{aligned} -1 &= 3k_1 + 3k_3 \\ 2 &= 7k_1 + 2k_2 + k_3 \\ 4 &= 4k_3 \\ 0 &= -4k_1 + 9k_2 + 5k_3 \end{aligned}$$

There is no solution for this system. Therefore $\vec{p}$ is not linear combination of $\vec{u}, \vec{v}$ and $\vec{w}$.

[3] 6. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vectors. Show that any linear combination of $\vec{u}, \vec{v}$ is also a linear combination of $-2\vec{u}$, $3\vec{v}$ and $\vec{w}$.

solution: Suppose that $\vec{x}$ is a linear combination of $\vec{u}$ and $\vec{v}$, i.e.,

$$\vec{x} = k_1\vec{u} + k_2\vec{v}$$

for some k_1 and k_2. Then

$$\vec{x} = c_1(-2\vec{u}) + c_2(3\vec{v}) + c_3\vec{w}$$

with

$$c_1 = \frac{k_1}{-2}, \quad c_2 = \frac{k_2}{3}, \quad c_3 = 0.$$

Therefore, $\vec{x}$ is also a linear combination of $-2\vec{u}$, $3\vec{v}$ and $\vec{w}$.

[6] 7. In each case, determine whether $\vec{u}$ is in the plane spanned by $\vec{v}$ and $\vec{w}$.

(a) $\vec{u} = \begin{bmatrix} -5 \\ 0 \\ 6 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \quad \vec{w} = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}$

(b) $\vec{u} = \begin{bmatrix} 6 \\ 7 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \quad \vec{w} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$

Solution: (a) Suppose that $\vec{u} = k_1\vec{v} + k_2\vec{w}$,i.e.,

$$\begin{cases} -5 = -k_1 + 3k_2 \\ 0 = 2k_1 + 4k_2 \\ 6 = 3k_1 \end{cases}$$

this system has a solution $k_1 = 2, k_2 = -1$. $\vec{u}$ is in the plane spanned by $\vec{v}$ and $\vec{w}$.

(b) Suppose that $\vec{u} = k_1\vec{v} + k_2\vec{w}$,i.e.,

$$\begin{cases} 6 = 3k_1 + 4k_2 \\ 7 = 5k_1 + 7k_2 \end{cases}.$$

We get $k_1 = 14,\ k_2 = -9$. $\vec{u}$ is in the plane spanned by $\vec{v}$ and $\vec{w}$.

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