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Existence of forced waves and gap formations for the lattice Lotka–Volterra competition system in a shifting environment

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ABSTRACT

We study the existence of forced traveling waves and gap formations for the lattice Lotka–Volterra competition system in a shifting habitat. By virtue of upper–lower solution method, we establish that the system admits a forced wave provided that the shifting speed of climate change falls in a certain interval. When the shifting speed is outside the range (except for the two endpoints), gap formations are then shown by theoretical proofs.

1. Introduction

In this paper we are concerned with the existence of forced traveling wave solutions and gap formations for the following lattice Lotka–Volterra competition system in a shifting habitat

$$\begin{cases} u_j'(t) = d_1\mathcal{D}_2[u_j](t) + u_j(t)[r_1(j-ct) - u_j(t) - v_j(t)], \\ v_j'(t) = d_2\mathcal{D}_2[v_j](t) + v_j(t)[r_2(j-ct) - v_j(t) - u_j(t)],\ t \in \mathbb{R}^+,\ j \in \mathbb{Z}, \end{cases} \tag{1.1}$$

where $\mathcal{D}_2[u_j](t) = u_{j+1}(t) - 2u_j(t) + u_{j-1}(t)$ and $\mathcal{D}_2[v_j](t) = v_{j+1}(t) - 2v_j(t) + v_{j-1}(t)$. Here, $u_j(t)$ and $v_j(t)$ stand for specific population densities at niches j and time t; $d_1 > 0$ and $d_2 > 0$ account for the diffusion coefficients; the constant $c \in \mathbb{R}$ can be understood as the shifting speed of the edge of the habitat; $r_1(\cdot)$ and $r_2(\cdot)$ represent the per capita growth rates. In this paper, they are assumed to satisfy the following hypothesis:

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(H1) $r_1(\cdot)$ is a continuous and decreasing function, and $r_2(\cdot)$ is a continuous and increasing function. Moreover, $r_i(\cdot), i = 1, 2$ satisfy the following asymptotic behaviors at infinity

$$r_1(-\infty) = K,\ r_1(\infty) = -L,\ \text{and}\ r_2(-\infty) = -L,\ r_2(\infty) = K, \tag{1.2}$$

where K, L are two positive numbers.

Recently, there is an increasing number of literatures concerning the impacts of climate change on dynamics of competitive species [1–3]. Biologically speaking, the surrounding environment or the spatial resource is changeable, and is subject to a shift with a constant speed. For instance, Hu and Li [4] studied the persistence and spreading speed for the following scalar lattice differential equation

$$u_t(t, x) = d[u(t, x + 1) - 2u(t, x) + u(t, x - 1)] + r(x - ct)u - u^2, \tag{1.3}$$

where $t > 0$, and $x \in \mathbb{Z}$ or $x \in \mathbb{R}$, by using the classical modified Bessel functions to the solutions of (1.3). For extensions of (1.3) in which the difference operator is replaced by a nonlocal dispersal operator, a differential operator, or an integral-difference formula, readers are further referred to [5–12] and the references therein.

System (1.1) can be viewed as a discrete version of the following competitive system

$$\begin{cases} u_t = d_1 u_{xx} + u(r_1(x - ct) - u - a_1 v), \\ v_t = d_2 v_{xx} + v(r_2(x - ct) - v - a_2 u),\ t \in \mathbb{R}^+,\ x \in \mathbb{R}, \end{cases} \tag{1.4}$$

with $a_1 = a_2 = 1$, which has been widely investigated recently. Under the hypothesis (H1), Berestycki et al. [13] proved the existence of a nontrivial forced wave of (1.4) in the special case when $c = 0$. Additionally, they also investigated the gap formation caused by the climate change in the case $c > c_{KPP}$, where $c_{KPP} = 2\sqrt{d_1 K}$ is the classical Fisher–KPP invasion speed of u at the far end when the species v is absent. Alternatively, by making a hypothesis that $r_i(\cdot), i = 1, 2$ share the same properties (see [14, Hypothesis 1] and also [15]), which is slightly different from (H1), Zhang et al. [14] established the coexistence and competitive exclusion of two competitors where the dynamics is completely different from our paper.

In this paper, we study the dynamics of (1.1) and establish two new results: (1) We prove the existence of forced traveling waves of system (1.1), connecting the two semi-trivial equilibria $(K, 0)$ and $(0, K)$, with the shifting speed in an interval $-\tilde{c}_0 < c < \bar{c}_0$ (see Theorem 2.1) for two positive constants $\tilde{c}_0$ and $\bar{c}_0$ that are biologically important (link to the KPP speed of a single species). This result includes the special case $c = 0$ where there exists a steady state solution satisfying the far end boundary conditions (1.2). (2) We rigorously show gap formations in two other cases: $c > \bar{c}_0$ and $c < -\tilde{c}_0$ (see Theorem 3.1).

2. Existence of forced waves of (1.1)

In this paper, a forced wave solution is referred to as a special solution in the form of

$$u_j(t) = U(z),\ v_j(t) = V(z), \tag{2.1}$$

with $z = j - ct$. Here, c is the same constant speed given in the reaction terms in (1.1). Biologically, this means that the invasion of the species can keep up the pace of the environment change. By substituting (2.1) into (1.1), we can get the following wave profile system

$$\begin{cases} d_1 \mathcal{D}_2[U] + cU' + U(r_1(z) - U - V) = 0, \\ d_2 \mathcal{D}_2[V] + cV' + V(r_2(z) - V - U) = 0, \quad z \in \mathbb{R}, \end{cases} \tag{2.2}$$

subject to the boundary conditions at infinity

$$U(-\infty) = K,\ U(\infty) = 0,\ \text{and}\ V(-\infty) = 0,\ V(\infty) = K. \tag{2.3}$$

Now, we get ready to state our first result regarding the existence of forced waves of system (1.1).

Theorem 2.1. *Assume that* $-\tilde{c}_0 < c < \bar{c}_0$ *with*

$$\tilde{c}_0 = \min_{\mu>0} \frac{d_2(e^{\mu} + e^{-\mu} - 2) + K}{\mu} > 0, \ \bar{c}_0 = \min_{\mu>0} \frac{d_1(e^{\mu} + e^{-\mu} - 2) + K}{\mu} > 0.$$

There exists a solution $(U(z), V(z))$ *to the system* (2.2) *satisfying* (2.3). *Furthermore,* $U(z)$ *and* $V(z)$ *are nonincreasing and nondecreasing functions respectively with respect to* $z \in \mathbb{R}$.

Proof. For clarity, we divide our proof into two steps.

Step 1. We intend to use the upper–lower solution method to prove two claims below.

Claim 1. Assume a monotonic and continuous function $a(z)$ satisfies the limits at infinity: $a(-\infty) = K,\ a(\infty) = -L - K$ and $c < c_0$ with

$$c_0 = \min_{\mu>0} \frac{d(e^{\mu} + e^{-\mu} - 2) + K}{\mu} > 0. \tag{2.4}$$

Then there exists a solution $w(z) > 0$ for the following boundary problem

$$\begin{cases} d\mathcal{D}_2[w(z)] + cw' + w(a(z) - w) = 0, \\ w(-\infty) = K,\ w(\infty) = 0. \end{cases} \tag{2.5}$$

Furthermore, $w(z)$ is nonincreasing in z. Since $w(z)$ is dependent on $a(z)$ and d, we may define $I(a, d)$ as the solution $w(z)$, i.e., $I(a, d) := w(a, d)(z)$. Then $I(a, d)$ is nondecreasing with respect to the function a.

Indeed, it is easy to check that the constant function $\overline{w} = K$ is an upper solution to the system (2.5). As for the construction of a lower solution, it is relatively difficult. Since $a(-\infty) = K$, there exist a z_0 and a small positive number $\epsilon > 0$ so that $a(z) \geq K - \epsilon$ for all $z \leq z_0$. We turn to consider the bistable wave profile equation as follows

$$d\mathcal{D}_2[\hat{w}(z)] + \hat{c}\hat{w}'(z) + f(\hat{w}(z)) = 0, \tag{2.6}$$

where

$$f(\hat{w}) = \begin{cases} \hat{w}(K - \epsilon - \hat{w}), & \hat{w} \geq 0, \\ \hat{w}(-\epsilon - \hat{w}), & \hat{w} < 0. \end{cases}$$

It can be derived from Theorem 3.5 of [16] that (2.6) possesses a decreasing bistable wave solution $\hat{w}$ with wave speed $\hat{c} = c_{\epsilon}$, satisfying

$$\hat{w}(-\infty) = K - \epsilon,\ \hat{w}(\infty) = -\epsilon. \tag{2.7}$$

Note c_{ϵ} is continuous with respect to ϵ. Letting $\epsilon \to 0^+$ in (2.6) leads to

$$d\mathcal{D}_2[\tilde{w}(z)] + \hat{c}\tilde{w}'(z) + \tilde{w}(K - \tilde{w}) = 0, \tag{2.8}$$

which admits non-negative traveling waves for $\hat{c} \geq c_0$, connecting K and 0, with the minimal speed c_0 given by (2.4). Clearly, for sufficiently small ϵ, the continuity of c_{ϵ} in ϵ gives that $c_{\epsilon} \to c_0$. For the same z_0 given above, we suppose that $\hat{w}(z) \geq 0$ for $z \leq z_0$ and $\hat{w}(z) < 0$ for $z > z_0$ due to the fact that a translation of $\hat{w}(z)$ is still a solution. Then we are able to give a lower solution to (2.5) with

$$\underline{w}(z) = \max\{\hat{w}(z), 0\}, \tag{2.9}$$

where $\hat{w}(z)$ is the solution of (2.6) satisfying (2.7). Precisely, when $\hat{w}(z) \leq 0$, $\underline{w}(z) = 0$, the required inequality for the lower solution follows naturally. When $\hat{w}(z) > 0$ (or equivalently, $z < z_0$), we have $\underline{w}(z) = \hat{w}(z)$. Plugging it into (2.5) with the assumption $c < c_0$ (because of $\hat{c} = c_{\epsilon} \to c_0$ as $\epsilon \to 0^+$) yields

$$d\mathcal{D}_2[\hat{w}(z)] + c\hat{w}' + \hat{w}(a(z) - \hat{w}) = (c - \hat{c})\hat{w}'(z) + \hat{w}(a(z) - (K - \epsilon)) \geq 0, \tag{2.10}$$

which implies that $\underline{w}(z)$ is a lower solution of (2.5) for $c < c_0$. By a procedure of upper–lower solution method, we can prove the existence of $w(z)$ to system (2.5) for $c < c_0$. As for the monotonicity of $I(a,d)$ in a, it is easy to see from the positivity coefficient $w(z)$ of $a(z)$.

Claim 2. Assume that a continuous and monotonic function $b(z)$ satisfies the limits at infinity: $b(-\infty) = -L-K,\ b(\infty) = K$ and $c > -c_0$. Then there exists a solution $\omega(z) > 0$ for the following boundary problem

$$\begin{cases} d\mathcal{D}_2[\omega(z)] + c\,\omega' + \omega(b(z) - \omega) = 0, \\ \omega(-\infty) = 0,\ \omega(\infty) = K. \end{cases} \tag{2.11}$$

Furthermore, $\omega(z)$ is nondecreasing. Since the proof of Claim 2 is similar to Claim 1, we omit it here. We mention here that we need the assumption $c > -c_0$ to complete the proof of claim 2.

Step 2. We define alternately two sequences of functions as follows:

$$\begin{aligned} &V_0 := 0,\, U_0 := I(r_1, d_1)\,, V_1 := I(r_2 - U_0, d_2), U_1 := I(r_1 - V_1, d_1) \ldots, \\ &V_{n+1} := I(r_2 - U_n, d_2),\, U_{n+1} := I(r_1 - V_{n+1}, d_1). \end{aligned} \tag{2.12}$$

It can be seen that $U_n(z)$ is nonincreasing and $V_n(z)$ is nondecreasing with respect to z for each $n \geq 1$, thanks to Claim 1 and Claim 2 respectively. On the other hand, we also can deduce from the monotonicity of $I(a,d)$ in a that $U_{n+1} \leq U_n,\ V_{n+1} \geq V_n,\ n \geq 0$. The boundedness of $\{U_n\}$ and $\{V_n\}$ ensures that there exist U and V so that $U_n \to U,\ V_n \to V$ pointwisely. In addition, it follows from the integral forms of

$$\begin{cases} -\,cU_n' = d_1\mathcal{D}_2[U_n] + U_n(r_1(z) - U_n - V_n), \\ -\,cV_{n+1}' = d_2\mathcal{D}_2[V_{n+1}] + V_{n+1}(r_2(z) - V_{n+1} - U_n), \end{cases} \tag{2.13}$$

that the pair of functions $(U,V)(z)$ is a C^1 solution to the system (2.2). Correspondingly, using Claims 1 and 2, we obtain the existence of U and V for $-\tilde{c}_0 < c < c_0$. Meanwhile, it is easy to see that $U(z)$ is nonincreasing and $V(z)$ is nondecreasing in z.

We are left to show that $(U,V)(z)$ satisfies the boundary conditions (2.3). Note that $U(z)$ and $V(z)$ are bounded and monotone, so the limits of them at infinity exist. We denote the limits by $U(\infty), U(-\infty), V(\infty)$ and $V(-\infty)$ respectively. Taking a look at (2.2), we obtain

$$\begin{aligned} &U(-\infty)(K - U(-\infty) - V(-\infty)) = 0,\ U(\infty)(-L - U(\infty) - V(\infty)) = 0, \\ &V(-\infty)(-L - U(-\infty) - V(-\infty)) = 0,\ V(\infty)(K - U(\infty) - V(\infty)) = 0, \end{aligned} \tag{2.14}$$

and have $U(\infty) = 0$ and $V(-\infty) = 0$. Moreover, we can derive $V(\infty) > 0$ by the monotonicity of the sequence V_n in n. As a result, from the last equation of (2.14), we have $V(\infty) = K$. As such, by $V_n \leq V$, we have $U_n \geq I(r_1 - V, d)$. This means $U(-\infty) = K$ from the first equality of (2.14). Hence, the proof is complete. ■

Remark 2.2. Claim 1 also holds if $a(\infty) = -L-K$ is replaced by $a(\infty) = -L$. This fact is needed in the construction of sequences defined in (2.12).

Remark 2.3. The existence or non-existence of forced wave of (2.2) in the critical cases $c = c_0$ or $c = -c_0$ remains open. Due to the fact that (2.2) is a coupled system, the ideas in [9, Theorem 2.1(i)] and/or [12, Theorem 1.2] cannot be directly applied. However, we still conjecture that, for the system (2.2), no forced traveling wave exists for the two critical cases mentioned above.

3. Gap formations

It is easy to verify that the function $f(\mu) = \frac{d_1(e^{\mu}+e^{-\mu}-2)+K}{\mu}$ is convex, and $f(\mu)$ can attain its minimal value at the unique critical point $\bar{\mu}$. In this subsection, we consider the system (1.1) subject to the following initial conditions

$$u(0,j)=u_0(j), \quad v(0,j)=v_0(j), \quad \text{for all } j\in\mathbb{Z}, \tag{3.1}$$

which are assumed to satisfy
(H2) $0\le u_0(j)\le K, 0\le v_0(j)\le K$ and the support of $u_0(j)$ is bounded from above. Namely, there exists an N so that $u_0(j)=0$ for all $j\ge N$.

We have the following gap formation result.

Theorem 3.1. *Assume $c>\bar{c}_0$ in (1.1) with initial data (3.1) satisfying (H2). We conclude that the unique bounded solution of (1.1) satisfies*

$$0\le u_j(t)\le K,\ 0\le v_j(t)\le K,\ \text{for all } t\ge 0,\ j\in\mathbb{Z}. \tag{3.2}$$

Moreover, for all c_1,c_2 satisfying $\bar{c}_0<c_1<c_2<c$, and for all constants $b_1,b_2\in\mathbb{R}$, we have

$$\sup_{j\ge c_1t+b_1} u_j(t)\le A_1e^{-\alpha_1 t},\ \forall t\ge 0, \tag{3.3}$$

and

$$\sup_{j\le c_2t+b_2} v_j(t)\le A_2e^{-\alpha_2 t},\ \forall t\ge 0, \tag{3.4}$$

where the constants A_1,α_1 are positive and are only dependent on the parameters $K,d_1,\ c_1,\ b_1,N$ (see (3.6)), the constants A_2,α_2 are positive and are only dependent on the parameters $K,L,d_2,c_2,\ b_2,\gamma,z_0$ (see (3.10)).

Proof. We firstly prove the estimate (3.3). Denote $\bar{u}=\bar{u}(t,j)$ as the solution of the following equation

$$u_j'(t)=d_1\mathcal{D}_2[u_j](t)+Ku_j(t), \tag{3.5}$$

subject to $\bar{u}(0,j)=u_0(j)$. One can verify directly that $w(t,j)=Ke^{-\bar{\mu}(j-\bar{c}_0t-N)}$ satisfies (3.5). As a result of $u_0(j)\le w(0,j)$ and the comparison principle, we have $u_j(t)\le\bar{u}(t,j)\le w(t,j)$. This in turn gives

$$\sup_{j\ge c_1t+b_1} u_j(t)\le A_1e^{-\alpha_1 t},\ \forall t\ge 0,$$

where

$$A_1=Ke^{-(b_1-N)\bar{\mu}},\quad \alpha_1=(c_1-\bar{c}_0)\bar{\mu}. \tag{3.6}$$

Thus, the proof of (3.3) is complete.

The proof of (3.4) is slightly different from the one of (3.3). Taking a view at the second equation of (1.1), it follows from the maximum principle that $v_j(t)\le K$ for all $t\ge 0,\ j\in\mathbb{Z}$. Thanks to the limit $\lim_{z\to-\infty}r_2(z)=-L$ and the monotonicity of $r_2(z)$, we know that there exists a number z_0 so that $r_2(z)\le-\eta$ (η satisfying $0<\eta<L$) for all $z\le z_0$. Consequently, we can construct an auxiliary equation

$$v_j'(t)-d_2\mathcal{D}_2[v_j](t)+\eta v_j(t)\le 0,\ j\le ct+z_0, \tag{3.7}$$

with $v_j(t)\le K$. Now, we are going to build up an upper solution to the following equation

$$v_j'(t)-d_2\mathcal{D}_2[v_j](t)+\eta v_j(t)=0.$$

To begin with, select two positive numbers γ,μ such that $-c\mu-d_2(e^{\mu}+e^{-\mu}-2)+\eta\ge 0$, and $0<\gamma\le\eta$. It is straightforward to check that the function defined as $\bar{v}_j(t)=K(e^{\mu(j-ct-z_0)}+e^{-\gamma t})$ satisfies

$$v_j'(t)-d_2\mathcal{D}_2[v_j](t)+\eta v_j(t)\ge 0.$$

This implies $\bar{v}_j(t)$ is an upper solution. Additionally, we have $v_j(t) \leq K < \bar{v}_j(t)$ for all $j = [ct + z_0]$, where $[\cdot]$ stands for the integer function. Summing up the above analysis, one can get the following inequalities

$$\begin{cases} (\bar{v}_j(t) - v_j(t))' - d_2\mathcal{D}_2[\bar{v}_j - v_j](t) + \eta(\bar{v}_j(t) - v_j(t)) \geq 0, \ j \leq ct + z_0, \\ \bar{v}_j(t) - v_j(t) > 0, \ \text{for all } t > 0, j = [ct + z_0], \\ \bar{v}_j(0) - v_j(0) > 0, \ j \leq z_0. \end{cases} \tag{3.8}$$

Again, by use of the comparison principle, one can conclude that $v_j(t) \leq \bar{v}_j(t)$ for all $j \leq ct + z_0$. In addition, it is easy to see that $c_2t + b_2 \leq ct + z_0$ holds for all $t > t_0$ by choosing t_0 sufficiently large. Therefore,

$$\sup_{j \leq c_2t+b_2} v_j(t) \leq \sup_{j \leq c_2t+b_2} K(e^{\mu(z-z_0)} + e^{-\gamma t}) \leq A_2e^{-\alpha_2 t}, \ \forall t \geq t_0, \tag{3.9}$$

where

$$A_2 = \max\{Ke^{(b_2-z_0)\mu}, K\}, \quad \alpha_2 = \min\{(c - c_2)\mu, \gamma\}. \tag{3.10}$$

To extend the range of t to $[0, t_0)$, it is only needed to increase suitably the constant A_2. As such, the proof of (3.3) is also complete. ■

Finally, we show that the gap formation described by (3.3) and (3.4) also exists for $c < -\tilde{c}_0$ with initial data (3.1) satisfying

(H3) $0 \leq u_0(j) \leq K, 0 \leq v_0(j) \leq K$ and the support of $v_0(j)$ is bounded from below. Namely, there exists N' so that $v_0(j) = 0$ for all $j \leq -N'$.

Theorem 3.2. *Assume $c < -\tilde{c}_0$ in (1.1) with initial data (3.1) satisfying (H3). We conclude that the unique bounded solution of (1.1) satisfies*

$$0 \leq u_j(t) \leq K, \ 0 \leq v_j(t) \leq K, \ \textit{for all } t \geq 0, \ j \in \mathbb{Z}.$$

Moreover, for all $\tilde{c}_1, \tilde{c}_2$ satisfying $c < \tilde{c}_1 < \tilde{c}_2 < -\tilde{c}_0$, and for all constants $\tilde{b}_1, \tilde{b}_2 \in \mathbb{R}$, there exist constants $\tilde{\alpha}_1, \tilde{\alpha}_2$ such that

$$\sup_{j \geq \tilde{c}_1t+\tilde{b}_1} u_j(t) \leq \tilde{A}_1e^{-\tilde{\alpha}_1 t}, \ \textit{and} \ \sup_{j \leq \tilde{c}_2t+\tilde{b}_2} v_j(t) \leq \tilde{A}_2e^{-\tilde{\alpha}_2 t}, \ \forall t \geq 0.$$

4. Conclusions and discussion

We have proved the existence of forced waves of (1.1) by means of two critical claims, plus an alternative iteration scheme. This enables us to establish new results for the wave speed in an interval. While the forced speed is beyond the interval (except for the two endpoints), it is interesting to show that gap formations appear as the consequence of climate change. It is worth mentioning that our method also can be used to study the continuous models [13] which we will further consider and extend in another paper (including the uniqueness and stability of the forced waves).

CRediT authorship contribution statement

Hongyong Wang: Writing - original draft, Formal analysis. **Chaohong Pan:** Investigation, Writing - review & editing. **Chunhua Ou:** Conceptualization, Project administration, Supervision.

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