## Assignment 5

Mathematics 2050
Fall 2018

Due: Oct 25,Thursday, 2018 . SHOW ALL WORK
[4] 1. Write down the $3 \times 2$ matrix $A$ with $a_{i j}=3 i j-\cos \frac{\pi j}{6}$.
[6] 2. Let $\mathbf{v}=\left[\begin{array}{l}1 \\ 2 \\ 3\end{array}\right]$ and $\mathbf{w}=\left[\begin{array}{r}0 \\ 4 \\ -5\end{array}\right]$. Let $A=\left(a_{i j}\right)=\left[\begin{array}{cc}\mathbf{v} & \mathbf{w} \\ \downarrow & \downarrow\end{array}\right]$ be the $3 \times 2$ matrix whose columns are $\mathbf{v}$ and $\mathbf{w}$ and let $B=\left(b_{i j}\right)=\left[\begin{array}{cl}\mathbf{v}^{T} & \rightarrow \\ \mathbf{w}^{T} & \rightarrow\end{array}\right]$ be the $2 \times 3$ matrix whose rows are $\mathbf{v}^{T}$ and $\mathbf{w}^{T}$. Find $a_{11}, a_{13}, a_{21}, b_{32}, b_{12}$, and $b_{22}$ if possible.
[4] 3. Express $\left[\begin{array}{r}-1 \\ 14 \\ 2\end{array}\right]$ as a linear combination of the columns of $A=\left[\begin{array}{lll}1 & 2 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 1\end{array}\right]$. Then find a vector x such $\mathrm{Ax}=\left[\begin{array}{r}-1 \\ 14 \\ 2\end{array}\right]$
[4] 4. Find the matrix $A$ if : $\left(3 A^{T}+3\left[\begin{array}{ll}1 & 0 \\ 0 & 2\end{array}\right]\right)^{T}=\left[\begin{array}{ll}8 & 0 \\ 3 & 1\end{array}\right]$
[6] 5. Compute the following matrix products.
(a) $\left[\begin{array}{rr}1 & 3 \\ 0 & -2\end{array}\right]\left[\begin{array}{rr}2 & -1 \\ 0 & 1\end{array}\right]$
(b) $\left[\begin{array}{rrr}5 & 0 & -7 \\ 1 & 5 & 9\end{array}\right]\left[\begin{array}{rrr}2 & 3 & 1 \\ 1 & 9 & 7 \\ -1 & 0 & 2\end{array}\right]$
[4] 6. Let $A=\left[\begin{array}{rr}2 & 5 \\ -3 & 1\end{array}\right]$ and $B=\left[\begin{array}{rr}4 & -5 \\ 3 & k\end{array}\right]$. What values of $k$, if any, will make $A B=B A$ ?
[4] 7. Let $A$ be a $2 \times 2$ matrix. If $A$ commutes with $\left[\begin{array}{ll}0 & 0 \\ 1 & 0\end{array}\right]$, show that $A=\left[\begin{array}{ll}a & 0 \\ c & a\end{array}\right]$ for some $a$ and $c$.
[4] 8. A matrix $P$ satisfies $P^{2}=P$. Suppose $4 Q=I-2 P$, where $I$ denotes the identity matrix with the size of $P$. What is $Q^{2}$ ?

