

Due: Thursday, Oct. 11

- [5] 1. Let $\vec{u} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$. Find the projection of \vec{u} onto \vec{v} ; and the projection of \vec{v} onto \vec{u} respectively.
- [10] 2. (a) Find two orthogonal vectors in the plane $x + y - 2z = 0$.
(b) Find the projection of $\vec{u} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ onto the plane $x + y - 2z = 0$.
- [5] 3. Calculate the distance from point $P(1, 2, 1)$ to the line $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 + t \\ 2 - 2t \\ 2t - 1 \end{bmatrix}$.
- [5] 4. Calculate the distance from point $P(1, 2, 1)$ to the plane $2x - y + 2z = 1$.
- [5] 5. Are vectors $\vec{u} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, and $\vec{x} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ linearly independent?
- [10] 6. Let $\vec{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.
(a) Show that \vec{u} , \vec{v} and \vec{w} are linearly independent.
(b) Show that any 3-dimensional vector $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ can be written as a linear combination of \vec{u} , \vec{v} and \vec{w} .
- [10] 7. Let $\vec{u} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$.
(a) Are \vec{u} , \vec{v} and \vec{w} linearly independent?
(b) Can any 3-dimensional vector be written as a linear combination of \vec{u} , \vec{v} and \vec{w} ?
If yes, please show your reason; if no, please provide one example.