MEMORIAL UNIVERSITY

DEPARTMENT OF MATH & STAT

ASSIGNMENT 3	**Math 2050**	FALL 2018

[3] 1. Find the equation of the plane that contains the point $Q(0,-1,0)$ and is parallel to the plane $2x+y-z=0$.

Solution: The normal vector is $\vec{n} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$. Hence, the equation of the plane is $2x+y-z = \vec{n}\cdot \vec{OQ} = -1$.

[3] 2. Find a vector of length 2 orthogonal to the plane $x+z=5$.

Solution: First we find a vector that is orthogonal to the plane. Note that the normal $\vec{n} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ is such a vector. The unit vector in the direction of $\vec{n}$ is $\vec{u} = \frac{\vec{n}}{||\vec{n}||} = \frac{1}{\sqrt{2}}\vec{n}$. Now, note that the vector $\frac{2}{\sqrt{2}}\vec{n}$ has length 2 and is orthogonal to the plane.

3. Consider the points A(0, 1, 0), B(-1, 0, 0) and C(0, 0, 1).

[5] (a) Find the equation of the plane passing through A, B, and C.

Solution: We have $\vec{AB} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$ and $\vec{AC} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$. A normal vector is $\vec{n} = \vec{AB}\times\vec{AC} = \begin{vmatrix} i & j & k \\ -1 & -1 & 0 \\ 0 & -1 & 1 \end{vmatrix} = -\vec{i}+\vec{j}+\vec{k} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$.

As a initial point we take $P_0 = A$. Then the equation of the plane is

$$-x+y+z = \vec{n}\cdot\vec{OA} = 1.$$

[3] (b) Find the area of the triangle ABC.

Solution: The area of the parallelogram with sides AB and AC is equal to

$$||\vec{AB}\times\vec{AC}|| = \sqrt{3}.$$

Hence, the area of the triangle ABC is $\frac{\sqrt{3}}{2}$.

[5] 4. Find the equation of the line of intersection of the planes $3x+y-2z=1$ and $x+2y-z=-3$.

Solution: We first find a direction vector $\vec{d}$ for the line of intersection. Let $\vec{n_1} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$ and $\vec{n_2} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ be normal vectors of the two planes. Now we look for a vector that is orthogonal to both n_1 and n_2. Since $\vec{d}$ is orthogonal to both n_1 and n_2, the natural candidate is

$\vec{d} = \vec{n_1} \times \vec{n_2} = \begin{vmatrix} i & j & k \\ 3 & 1 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 3\vec{i} + \vec{j} + 5\vec{k} = \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}$. Now among the many points on the intersection, we need one of them. We let $z = 0$ in the equation of the planes and get that $3x + y = 1$ and $x + 2y = -3$. Solving these equations, we get $x = 1$ and $y = -2$. Hence, $(1, -2, 0)$ is on the line and now we can write the equation of the line as

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + t \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}.$$

[5] 5. Find all points (if any) of the intersection of the plane $x - y + z = 1$ and the line $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

Solution: Suppose that there is a point (x, y, z) in the intersection of them. From the equation of the line, we have $x = t, y = 3, z = -t$. Now we substitute these values into the equation of the plane and get: $(t) - (3) + (-t) = 1$. We simplify and get $-3 = 1$ which is not possible. It means that there is no point in the intersection of the plane and the line.

6. Consider the lines with equations $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

[1] (a) Are the lines parallel? justify your answer.

Solution: The lines are not parallel because their direction vectors $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ are not parallel.

[5] (b) Determine whether or not the lines intersect. Find the point of intersection (if it exists).

Solution: If the lines intersect, say at a point (x, y, z), then (x, y, z) is on the both lines. So, there exist s, t such that

$$x = 1 + t = 1 + s$$
$$y = -1 = -s$$
$$z = 1 = s$$

From Eqns 2 and 3 equation, we get $s = 1$. From Eqn 1, we get $t = 1$. Now we can sub in either $t = 1$ into first equation or $s = 1$ into second equation and see that $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ is the point of intersection.

[5] 7. Let $\mathbf{u} = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. Compute $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$ and $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$. Should the answers be the same?

Solution: We have $\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ -2 & 3 & 0 \\ 1 & 2 & 3 \end{vmatrix} = \begin{bmatrix} 9 \\ 6 \\ -7 \end{bmatrix}$ and

$(\vec{u} \times \vec{v}) \times \vec{w} = \begin{vmatrix} i & j & k \\ 9 & 6 & -7 \\ 0 & 0 & 1 \end{vmatrix} = \begin{bmatrix} 6 \\ -9 \\ 0 \end{bmatrix}$. Also, $\vec{v} \times w = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 0 & 0 & 1 \end{vmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$ and

$\vec{u} \times (\vec{v} \times \vec{w}) = \begin{vmatrix} i & j & k \\ -2 & 3 & 0 \\ 2 & -1 & 0 \end{vmatrix} = \begin{bmatrix} 0 \\ 0 \\ -4 \end{bmatrix}$.

We deduce that $(\vec{u} \times \vec{v}) \times \vec{w} \neq \vec{u} \times (\vec{v} \times \vec{w})$. This means that the cross product in general is not associative.

[5] 8. Find the equation of the plane containing both lines $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

Solution: We need to find a normal vector $\vec{n}$ for the plane. Note that the points $P = (1, -1, 1)$ and $Q = (1, 0, 0)$ are on the lines and so P and Q are also on the plane. Hence, the vector $\vec{PQ}$ is also on the plane. Now $\vec{PQ} = Q - P = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$.

Also, the direction vector $\vec{d} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is parallel to the plane. Hence, we can take $\vec{n} = \vec{PQ} \times \vec{d} = \begin{vmatrix} i & j & k \\ 0 & 1 & -1 \\ 1 & 0 & 0 \end{vmatrix} = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}$. Now, we can use $n = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}$ and the point $Q = (1, 0, 0)$ to write the equation of the plane:

$$y + z = 0.$$

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