Symmetry, Invariants, and their Applications: A Celebration Peter Olver's 70th Birthday

August 3–5, 2022
Dalhousie University, Halifax, Nova Scotia, Canada

Participants

William Dylan Adams
Stephen Anco
Amine Bahayou
Gavin Ball
Gloria Marí Beffa
Alex Bihlo
Nicoleta Bila
George Bluman
Mireille Boutin
Dario Brooks
Louis Bu
Joseph Burdis
Elsa Cardoso-Bihlo
Roman Cherniha
Sehun Chun
Jeanne Clelland
Alan Coley
Amanullah Dar
Priscila Leal da Silva
Marek Elzanowski
Suresh Eswarathasan
Mark Fels
Rui Loja Fernandes
Igor Leite Freire
Daniel Presciosco Garcelán
Ian George
Artemio González-López
Illia Hayes
Peter Hydon
Farheen Ibraheem
Adenike Florence Idowu
Thomas Ivey
Niky Kamran
Taylor J. Klotz
Irina Kogan
Serhii Koval
Boris Kruglikov
Alexandre Landry
Melvin Leok
Debra Lewis
Jean-Pierre Magnot
Vaughn Menchions
Robert Milson
Hanlin Mo
Peter J. Olver
Riley O'Neill
Linyu Peng
Dmytro Popovych
Roman Popovych
Changzheng Qu
Greg Reid
Peter Rock
Andy Sageman-Furnas
Eivind Schneider
Artur Sergyeyev
Cheri Shakiban
Ekaterina Shemyakova
Alexey Shevyakov
Komal Singla
Roman Smirnov
Allen Tannenbaum
Keti Tenenblat
Jonathan Tot
Brian Tran
Francis Valiquette
Joel Villatoro
Lewis White
Thomas Wolf

Wednesday August 3

(Talks are in the Chase Building, room 319 – Colloquium Room)
⋆ = Online speaker
Contact one of the organizers for the Zoom information

Time	Speaker	Title
8:45 – 9:00	Jason Brown	WELCOME AND OPENING REMARKS
9:00 – 9:45	Thomas Ivey	TUTORIAL Introduction to classical moving frames
9:50 – 10:10	Taylor J. Klotz	Symmetry and geometry of explicitly integrable control systems
10:15 – 10:45	**COFFEE BREAK**	
10:45 – 11:05	Debra Lewis⋆	Symmetries and invariants in behavioral optimal control
11:10 – 11:30	Roman Smirnov	Bowley's law revisited
11:35 – 13:30	**LUNCH**	
13:30 – 14:15	Mireille Boutin	TUTORIAL Object Recognition in Practice: An introduction
14:20 – 14:40	Allen Tannenbaum⋆	Geometric Analysis of Networks and Genomics
14:45 – 15:15	**COFFEE BREAK**	
15:15 – 15:35	Melvin Leok	Variational accelerated optimization on Riemannian manifolds
15:40 – 16:00	Sehun Chun	Finding the "geometry" of the heart and brain with moving frames
Evening Activity	Ferry (Halifax Transit) – Dartmouth Restaurants	

Thursday August 4

(Talks are in the Chase Building, room 319 – Colloquium Room)

⋆ = Online speaker

Contact one of the organizers for the Zoom information

Time	Speaker	Title
9:00 – 9:45	Gloria Marí Beffa	TUTORIAL From geometric invariants to integrable systems via Lie groups
9:50 – 10:10	Changzheng Qu⋆	Integrable systems and invariant geometric flows in affine-related geometries
10:15 – 10:45	**COFFEE BREAK**	
10:45 – 11:05	Artur Sergyeyev⋆	From contact geometry to multidimensional integrable systems
11:10 – 11:30	Keti Tenenblat⋆	On self-similar solutions to the curvature flow for curves
11:35 – 13:30	**LUNCH**	
13:30 – 14:15	Alan Coley	TUTORIAL Scalar polynomial invariants and applications
14:20 – 14:40	Artemio González-López⋆	Inhomogeneous XX spin chains and QES models
14:45 – 15:15	**COFFEE BREAK**	
15:15 – 15:35	Niky Kamran⋆	Ramified local isometric embeddings of singular Riemannian metrics
15:40 – 16:00	Andy Sageman-Furnas	Constructing isometric tori with the same curvatures
17:30 – 19:00	Posters, Discussion & Reception (Atlantica Hotel) Gavin Ball, Dario Brooks, Lewis White, Dmytro Popovych, Joel Villatoro, Brian Tran, Nicoleta Bila	
19:00 – 21:00	Dinner (Atlantica Hotel)	

Friday August 5

(Talks are in the Chase Building, room 319 – Colloquium Room)

⋆ = Online speaker

Contact one of the organizers for the Zoom information

Time	Speaker	Title
9:00 – 9:45	Rui Loja Fernandes	TUTORIAL The geometry of Cartan's realization problems
9:50 – 10:10	Jean-Pierre Magnot⋆	On symmetries of weak solutions: A rigorous approach by diffeologies
10:15 – 10:45	**COFFEE BREAK**	
10:45 – 11:05	Boris Kruglikov⋆	ODEs with non fiber-preserving symmetry
11:10 – 11:30	Peter Hydon⋆	Conservation laws and symmetries that depend on arbitrary functions
11:35 – 13:30	**LUNCH**	
13:30 – 13:50	Roman Cherniha⋆	Symmetries and exact solutions of the diffusive Lotka–Volterra system
13:55 – 14:15	Stephen Anco⋆	Bracket structure for adjoint-symmetries
14:20 – 14:40	Alexey Shevyakov	Two approximate symmetry frameworks for nonlinear partial differential equations with a small parameter: Comparisons, relations, approximate solutions
14:45 – 15:15	**COFFEE BREAK**	
15:15 – 15:35	Thomas Wolf	Solutions of a coupled KdV-NLS system
15:40 – 16:00	Serhii Koval	On structures of the complete point symmetry group and the maximal Lie invariance algebra of an ultraparabolic Fokker–Plank equation
17:00 – 19:30	Excursion (Herring Cove Provincial Park Reserve or York Redoubt National Historic Site)	

Abstracts

Thomas Ivey (College of Charleston)

Introduction to classical moving frames

The moving frame (*repère mobile*) was introduced by Élie Cartan as a way of simplifying the derivation of geometric invariants and solving equivalence problems. It constitutes a uniform approach that can be adapted for a variety of geometries defined by transitive Lie group actions. Simple examples, such as curves in the Euclidean plane, the projective plane and the conformal 3-sphere, will be discussed.

Taylor J. Klotz (University of Hawaii)

Symmetry and geometry of explicitly integrable control systems

Linearizable control systems have been well studied from both the perspective of specific applications and also geometry and symmetry. The celebrated G.S. algorithm (developed by Gardner and Shadwick and earlier work with Wilkens) yields both a test for a control system to be linearizable as well as producing the linearizing coordinates belonging to a class of diffeomorphisms called static feedback transformations (SFTs). Such control systems admit explicit formulas for generic trajectories in terms of arbitrary functions and their derivatives with no quadrature needed. It is interesting to ask about the class of control systems which are provably non-linearizable (i.e. intrinsically nonlinear) but still have such a convenient description of their generic trajectories. Important progress has been made in case the control system under consideration has two types of linearizability with respect to local finite dimensional subgroups of the pseudo-group of SFTs. This concept, known as cascade feedback linearizability, was introduced by Vassiliou. I'll present an overview of such systems, recent progress including real-world models, and finish on some open related problems.

Debra Lewis (UC Santa Cruz)

Symmetries and invariants in behavioral optimal control

The importance of symmetries in Hamiltonian mechanics and optimal control theory for physical systems has long been recognized; techniques for recognizing and exploiting symmetries in optimal control models arising in from the behavioral sciences are less well developed. We seek models and metrics that reflect the diversity of the populations under consideration, with costs depending on invariants that provide a flexible measure of success and/or efficacy.

Mireille Boutin (Purdue University)

Object Recognition in Practice: An introduction

This tutorial will focus on the general problem of assigning a category (e.g. an object type) to a noisy observation. We will describe the probabilistic formulation of the problem, where the object categorization is viewed as an estimation problem, and relate it to machine learning methods. Scenarios where the underlying framework is not applicable will be highlighted. Along the way, we will show how invariant theory complements data-driven object recognition approaches, and why work in the field is still relevant despite the significant advances in training-based numerical methods.

Allen Tannenbaum (Stony Brook University)

Geometric Analysis of Networks and Genomics

We will describe some recent work using discrete notions of curvature to study biological networks with applications to oncology.

Melvin Leok (University of California, San Diego)

Variational accelerated optimization on Riemannian manifolds

Efficient optimization has become one of the major concerns in data analysis. There has been a lot of focus on first-order optimization algorithms because of their low cost per iteration. In 1983, Nesterov's Accelerated Gradient method (NAG) was shown to converge in $\mathcal{O}(1/k^2)$ to the minimum of the convex objective function f, improving on the $\mathcal{O}(1/k)$ convergence rate exhibited by the standard gradient descent methods, which is the phenomenon referred to as acceleration. It was shown that NAG limits to a second order ODE, as the time step goes to 0, and that the objective function $f(x(t))$ converges to its optimal value at a rate of $\mathcal{O}(1/t^2)$ along the trajectories of this ODE. In this talk, we will discuss how the convergence of $f(x(t))$ can be accelerated in continuous time to an arbitrary convergence rate $\mathcal{O}(1/t^p)$ in normed spaces, by considering flow maps generated by a family of time-dependent Bregman Lagrangian and Hamiltonian systems which is closed under time rescaling. We will then discuss how this variational framework can be exploited together with the time-invariance property of the family of Bregman Lagrangians using adaptive geometric integrators to design efficient explicit algorithms for symplectic accelerated optimization based on time-adaptive discrete Hamiltonian variational integrators. Finally, we will discuss briefly the generalization from normed spaces to Riemannian manifolds.

Sehun Chun (Yonsei Univeristy)

Finding the "geometry" of the heart and brain with moving frames

For the biological electric propagation in the heart and brain, the geometry of the propagation represents the source-sink relation between excited and excitable tissues. Thus, geometry often delivers a similar meaning as the connection. In the heart, the cardiac connection corresponds to the alignment of the cardiac fiber with strong anisotropic conductivity. In the brain, the neural connection corresponds to the configuration of the neural network and its strength. However, these two connections are different. The heart's connection is naturally set up from birth, whereas the brain's connection changes according to the human's learning and adaptation. Nevertheless, both organs' connectivity configuration and change over time are the critical factors in understanding the basic mechanisms of their miraculous functions and studying related pathologies, particularly heart arrhythmia and brain dementia.

To apply moving frames to both connections, an orthonormal set of basis vectors, or moving frames, is constructed at each grid point of the domain. The first tangent vector is aligned with the electric propagation direction, and the remaining two unit tangent vectors are readjusted accordingly. One critical product of moving frames is the Riemann curvature of the orthonormal basis for the prediction of conduction failure. The conduction failure in the heart and brain can be related to the cause of arrhythmia and dementia because a component of the Riemann curvature tensor reflects the source and sink match between excited and excitable tissues. In this talk, I will explain the past development of moving frames for the geometric analysis of the heart and brain and how it provided new insight into their propagation mechanism. Furthermore, the future development of moving frames is suggested considering the realistic physiological domain of the heart and brain. The first is to represent the bidomain by moving frames. At least two sets of moving frames are constructed at one grid point because the biological domain consists of the intercellular space (cell) and interstitial space (salty water) everywhere. The second is the representation of the time variable by moving frames considering the time-dependent variation of connectivity in the heart and brain.

Gloria Marí Beffa (University of Wisconsin–Madison)

From geometric invariants to integrable systems via Lie groups

In this talk we will start by comparing the role of the Lie group in generating invariants of curves and polygons in a geometric setting to that of the Lie group generating Hamiltonian structures via Poisson Lie groups. We will then connect them both directly, giving a brief summary of current advances in integrable systems that are based on what one can learn from geometric settings.

Changzheng Qu (Ningbo University)

Integrable systems and invariant geometric flows in affine-related geometries

It is well-known that integrable systems are related to invariant geometric flows in certain geometries. In this talk, we are mainly concerned with invariant geometric flows in affine-related geometries including centro-equiaffine, centro-affine, affine and affine-symplectic geometries. First, we show that the specific invariant geometric flows in those geometries are related respectively to well-known integrable systems. Second, the geometric formulations to integrability features of the resulting systems are investigated. Third, the geometric formulations of Miura-transformation and its various extensions are also investigated. This talk is based on the works joint with Peter Olver, Kaiseng Chou, Yun Yang and Zhiwei Wu.

Artur Sergyeyev (Silesian University in Opava)

From contact geometry to multidimensional integrable systems

Contact geometry is well known to play an important role in the geometric approach to the study of partial differential systems. In this talk we showcase a novel application of three-dimensional contact geometry, where it helps answering a longstanding question of just how exceptional are partial differential systems in four independent variables that are integrable in the sense of soliton theory. It turns out that such systems are far more numerous than it was believed, and we provide an effective explicit construction, involving contact vector fields, for a large class of systems in question along with their Lax pairs. As a byproduct, we present a first example of an integrable partial differential system in four independent variables with a nonisospectral Lax pair which is algebraic, rather than rational, in the spectral parameter.

Keti Tenenblat (Universidade de Brasília)

On self-similar solutions to the curvature flow for curves

I will present results on the soliton solutions to the curve shortening flow on the 2-dimensional unit sphere and on the 2-dimensional hyperbolic space. I will also discuss recent results on the self-similar solutions to the curvature flow and to the inverse curvature flow, for curves on the 2-dimensional light cone. In each case, the geometry of the curves will be described. The talk is based on my recent papers in collaboration with Hiuri F. Dos Reis and Fabio N. Da Silva.

Alan Coley (Dalhousie University)

Scalar polynomial invariants and applications

Scalar curvature invariants are scalars constructed from the Riemann tensor and its covariant derivatives. I shall briefly discuss some properties of scalar polynomial curvature invariants in Lorentzian manifolds and present some applications in general relativity.

Artemio González-López (Universidad Complutense de Madrid)

Inhomogeneous XX spin chains and QES models

I will show how inhomogeneous XX spin chains (or free fermion systems with nearest-neighbors hopping) can be related to certain QES models on the line giving rise to a family of weakly orthogonal polynomials. I will present a complete classification of all such models, which include two families related to the Lamé (finite gap) quantum potential on the line, and their associated XX chains. Time permitting, I will explain how to derive an asymptotic approximation to the Rényi entanglement entropy at half filling of one of these Lamé chains by studying its continuum limit, which turns out to describe a massless Dirac fermion on a suitably curved background.

Niky Kamran (McGill University)

Ramified local isometric embeddings of singular Riemannian metrics

We shall discuss the existence of local isometric embeddings into Euclidean space for analytic Riemannian metrics g defined on a domain $U \subset \mathbb{R}^n$, which are singular in the sense that the determinant of the metric tensor is allowed to vanish in a rather precise sense at an isolated point. The metrics being thus considered are degenerate, but in a fairly mild sense and in exactly one direction. Specifically, we show that under suitable technical assumptions, there exists a local analytic isometric embedding $\boldsymbol{u}$ from $(U', \Pi^* g)$ into Euclidean space $\mathbb{E}^{(n^2+3n-4)/2}$, where $\Pi : U' \to U \backslash \{0\}$ is a finite Riemannian branched cover of a deleted neighborhood of the origin. Our result can thus be thought of as a generalization of the classical Cartan-Janet Theorem to the singular setting in which the metric tensor is degenerate at an isolated point. Our proof uses Leray's ramified Cauchy-Kovalevskaya Theorem for analytic differential systems, in the form established by Choquet-Bruhat for non-linear systems. This is joint work with Alberto Enciso (ICMAT, Madrid).

Andy Sageman-Furnas (North Carolina State University)

Constructing isometric tori with the same curvatures

Which data determine an immersed surface in Euclidean three-space up to rigid motion? A generic surface is locally determined by only a metric and mean curvature function. However, there are exceptions. These may arise in a family like the isometric family of vanishing mean curvature surfaces transforming a catenoid into a helicoid, or as a so-called Bonnet pair of surfaces. For compact surfaces, Lawson and Tribuzy proved in 1981 that a metric and non-constant mean curvature function determine at most one immersion with genus zero, but at most two compact immersions (compact Bonnet pairs) for higher genus. This led them to ask if compact Bonnet pairs exist. In this talk, we discuss our recent construction of the first examples of compact Bonnet pairs. It uses a local classification by Kamberov, Pedit, and Pinkall in terms of isothermic surfaces. Moreover, we describe how a structure-preserving discrete theory for isothermic surfaces and Bonnet pairs led to this discovery. The smooth theory is joint work with Alexander Bobenko and Tim Hoffmann and the discrete theory is joint work with Tim Hoffmann and Max Wardetzky.

Gavin Ball (University of Wisconsin, Madison)

Associative submanifolds in some nearly parallel G_2-manifolds

A nearly parallel G_2-structure is the natural geometric structure induced on the seven-dimensional link of a conical manifold with holonomy Spin(7). The link of a conical Cayley submanifold gives an associative submanifold of the nearly parallel G_2-manifold, and thus associative submanifolds of nearly parallel G_2-manifolds provide models for conically singular Cayley submanifolds. I will explain a construction of certain associative submanifolds in two settings: in the Berger space SO(5)/SO(3) with its homogenous nearly parallel G_2-structure, and in squashed 3-Sasakian manifolds. In both cases the submanifolds are ruled by a special class of geodesics and arise from a construction based on holomorphic curves in the spaces of rulings. This is joint work with Jesse Madnick.

Dario Brooks (Dalhousie University)

Geometric horizons and scalar curvature invariants

In general relativity, it is known that the event horizon of a stationary black hole can be located using scalar polynomial curvature invari- ants (SPIs) which are special in that they vanish on the horizon. Of course, realistic black holes undergo evolutionary processes and are thus dynamical. We will discuss the concept of a geometric horizon, an analogue to the stationary horizon to be associated with more general, dynamic black hole geometries, and consider the imploding spherically symmetric metrics to illustrate how a geometric horizon can be detected using SPIs.

Lewis White (University of Kent)

Partial difference moving frames

The poster will present the most recent results of applying moving frames to the difference calculus of variations. The method of moving frames is used here to calculate the difference Euler–Lagrange equations directly in terms of the invariants. An example is given to illustrate this theory.

Dmytro Popovych (Memorial University of Newfoundland)

Flags of subalgebras under contractions of Lie algebras

Contractions of Lie algebras is a kind of limit transition from the initial Lie algebra to the contracted one. Existence of a contraction can be demonstrated constructively, and impossibility can be proven using different criteria, or directly. We present a new broad criterion from which some commonly used criteria can be derived, and demonstrate its practical use by proving nonexistence of contractions in a number of pairs of 6d nilpotent real Lie algebras.

Joel Villatoro (Washington University in St. Louis)

Singular Lie groups associated to sheaves of Lie algebras

In this poster we will look at some homotopy theoretic procedures for constructing symmetry groups associated to certain Lie algebras. We will also discuss their structure.

Brian Tran (University of California, San Diego)

Geometric Methods for Adjoint Systems

Adjoint systems are widely used to inform control, optimization, and design in systems described by ordinary differential equations or differential-algebraic equations. We explore the geometric properties and develop methods for such adjoint systems. In particular, we utilize symplectic and presymplectic geometry to investigate the properties of adjoint systems associated with ordinary differential equations and differential-algebraic equations, respectively. We show that the adjoint variational quadratic conservation laws, which are key to adjoint sensitivity analysis, arise from (pre)symplecticity of such adjoint systems. We discuss various additional geometric properties of adjoint systems, such as symmetries and variational characterizations. For adjoint systems associated with a differential-algebraic equation, we relate the index of the differential-algebraic equation to the presymplectic constraint algorithm of Gotay et al. [Presymplectic manifolds and the Dirac–Bergmann theory of constraints, *J. Math. Phys.* **19** (1978), 2388–2399]. As an application of this geometric framework, we discuss how the adjoint variational quadratic conservation laws can be used to compute sensitivities of terminal or running cost functions. Furthermore, we develop structure-preserving numerical methods for such systems using Galerkin Hamiltonian variational integrators (Leok and Zhang [Discrete Hamiltonian variational integrators, *IMA J. Numer. Anal.* **31** (2011), 1497–1532]) which admit discrete analogues of these quadratic conservation laws. We additionally show that such methods are natural, in the sense that reduction, forming the adjoint system, and discretization all commute, for suitable choices of these processes. We utilize this naturality to derive a variational error analysis result for the presymplectic variational integrator that we use to discretize the adjoint DAE system. Finally, we discuss the application of adjoint systems in the context of optimal control problems, where we prove a similar naturality result.

Nicoleta Bila (Fayetteville State University)

Special symmetry reductions for Tzitzeica surfaces PDE

One of the most prestigious Romanian mathematicians, Gheorghe Tzitzeica (1873-1939), introduced two centro-affine invariants that carry his name today. A Tzitzeica curve is a spatial curve for which the ratio of its torsion and the square of the distance from the origin to the osculating plane at an arbitrary point of the curve is constant. A Tzitzeica surface is a surface for which the ratio of its Gaussian curvature and the fourth power of the distance from the origin to the tangent plane at any arbitrary point of the surface is constant. It may be shown that the asymptotic curves on a Tzitzeica surface with negative Gaussian curvature are Tzitzeica curves. The differential equations related to the Tzitzeica curves and Tzitzeica surfaces, respectively, are both nonlinear and challenging to solve. In this work, it is shown how one can use the Tzitzieca curves as symmetry reductions for the nonlinear Tzitzeica surface partial differential equation. The method may be further explored in using special curves on a surface as symmetry reductions for the related surface's partial differential equation.

Rui Loja Fernandes (UIUC)

The geometry of Cartan's realization problems

Applying Lie groupoid theory, one can give a description of an important class of Cartan's realization problems (the ones that have a "finite dimensional" space of invariants) and their solutions. The talk is based on the preprint arXiv:1907.13614, joint with Ivan Struchiner.

Jean-Pierre Magnot (Université d'Angers)

On symmetries of weak solutions: A rigorous approach by diffeologies

We describe here a class of symmetries (global or infinitesimal) and of invariants for weak solutions, that is, solutions constructed through sequences of approximate solutions of a functional equation. We present here published and pre-published results.

Boris Kruglikov (UiT the Arctic University of Norway)

ODEs with non fiber-preserving symmetry

Scalar ODEs with essentially contact symmetries and ODE systems with essentially point symmetries are rare among all symmetric differential equations. I will describe those using the technique of absolute, relative and conditional differential invariants. The talk is joint with Eivind Schneider (UiT) based on arXiv:2202.10331.

Peter Hydon (University of Kent)

Conservation laws and symmetries that depend on arbitrary functions

Emmy Noether's Second Theorem links variational symmetries that depend on arbitrary functions of all (that is, p) independent variables to differential relations between the Euler-Lagrange equations. Recently, this was extended by Peter Olver to functions of p jet space variables, and by Elizabeth Mansfield and the first author to functions of fewer than p variables. For systems that are not variational, however, relatively little is known about symmetries and conservation laws that depend on arbitrary functions. This talk introduces some recent results and applications. (Joint work with John King)

Roman Cherniha (Institute of Mathematics, NAS of Ukraine)

Symmetries and exact solutions of the diffusive Lotka–Volterra system

The talk is devoted to the search for exact solutions of the diffusive Lotka-Volterra systems and their applications. The symmetry-based methods (especialy Lie and conditional symmetries) are used for these purposes. Because the diffusive Lotka-Volterra systems are used for mathematical modeling enormous variety of processes in ecology, biology, medicine, physics and chemistry, the talk should be interesting for a wide audience of researchers. This is joint work with Vasyl' Davydovych (Institute of Mathematics, NAS of Ukraine, Kyiv).

Stephen Anco (Brock University)

Bracket structure for adjoint-symmetries

A natural bracket structure has been uncovered for adjoint-symmetries. It has connections to symmetry Lie bracket, Hamiltonian and recursion operators, and conservation laws.

Alexey Shevyakov (University of Saskatchewan)

Two approximate symmetry frameworks for nonlinear partial differential equations with a small parameter: Comparisons, relations, approximate solutions

The frameworks of Baikov-Gazizov-Ibragimov (BGI) and Fushchich-Shtelen (FS) approximate symmetries are used to study symmetry properties of partial differential equations with a small parameter. In general, it is shown that unlike in the ordinary differential equation (ODE) case, unstable BGI point symmetries of unperturbed partial differential equations (PDE) do not necessarily yield local approximate symmetries for the perturbed model. While some relations between the BGI and FS approaches can be established, the two methods yield different approximate symmetry classifications. Detailed classifications are presented for two nonlinear PDE families. The second family includes a one-dimensional wave equation describing the wave motion in a hyperelastic material with a single family of fibers. For this model, approximate symmetries can be used to compute approximate closed-form solutions. Wave breaking times are found numerically and using the approximate solutions, and yield comparable results. This is a joint work with Mahmood Tarayrah and Brian Pitzel.

Roman Smirnov (Dalhousie University)

Bowley's law revisited

Bowley's Law is a stylized fact of economics stating that the share of national income paid out to the employees as compensation for their work (i.e., the wage or labor share) remains unchanged over time. The economic data collected in different countries from the end of the 19th century until about 1980 gave rise to and strongly supported this law, which was widely accepted by the economics community at the time. This law is now subject to doubt, however, as recent data patterns appear to deviate from it.

We present a mathematical model demonstrating that the wage share can be treated as a time-independent invariant under certain conditions, thus defining the limitations of Bowley's Law. This is a joint work with Kunpeng Wang (Sichuan University-Pittsburgh Institute).

Thomas Wolf (Brock University)

Solutions of a coupled KdV-NLS system

The talk reports about integrability investigations of a coupled gKdV-NLS system consisting of a real and complex equation for a real and a complex function. The system involves a free parametric exponent and two parametric signs.

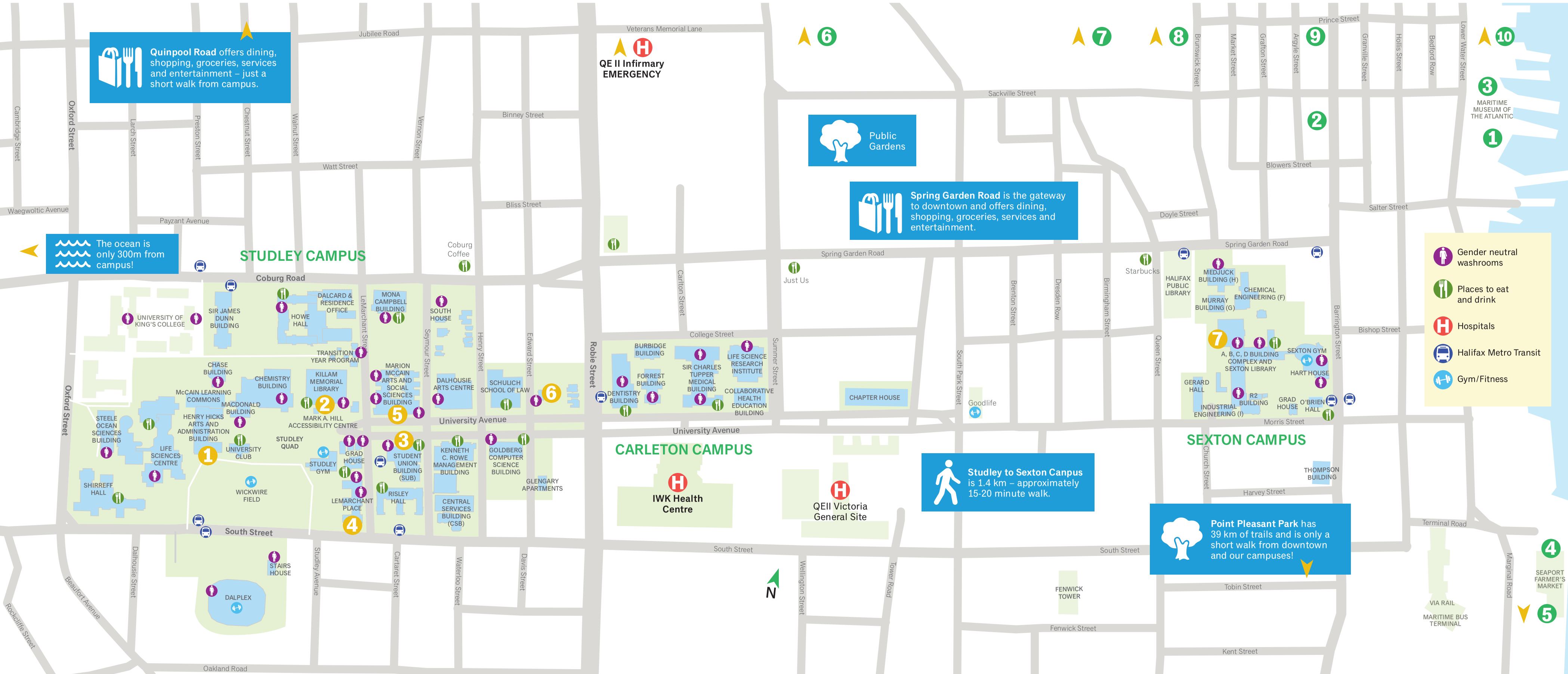
In the first part all point symmetries and conservation laws with conserved densities of differential order at most 1 are determined.

In the second part the ODE system for frequency-modulated traveling waves is investigated. A novel integration method is used to obtain exact solutions, including cases for which no solutions were previously known.

Serhii Koval (Memorial University of Newfoundland)

On structures of the complete point symmetry group and the maximal Lie invariance algebra of an ultraparabolic Fokker–Plank equation

In the process of carrying out the extended symmetry analysis of an ultraparabolic Fokker–Planck equation with three independent variables, which is also called the Kolmogorov equation and is singled out within the class of such Fokker–Planck equations by its remarkable symmetry properties, we compute the complete point symmetry group of the Kolmogorov equation using the direct method, analyze its structure and single out its essential subgroup and study its representation. Using the representation of this group on its algebra we successfully list inequivalent one- and two-dimensional subalgebras of the essential and maximal Lie invariance algebras of this equation.



DALHOUSIE

1 Henry Hicks
- Registrar's Office - Student Service Centre
- Student Accounts
- Office of Human Rights, Equity & Harassment Prevention

2 Killam Library
- Advising and Access Services
- First-year Advising
- Writing Centre
- Dal Ally Peer Advisor
- IT help desk
- Hardware Services
- Learning Commons (study space and computer access)

3 Student Union Building
- DSU offices
- DSU Equity & Accessibility Office
- DSU Wellness room
- DSU Food bank
- Dalhousie Bookstore
- Career & Leadership Development Centre
- Off-Campus Student Lounge
- CKDU radio
- DSU Loaded Ladle (free campus eats)
- The Grawood Bar & Grill
- Campus Copy
- Student Dispute Resolution Office
- Native Education Counselling Unit

4 LeMarchant Place
- Student Health Services Clinic
- Student Health Promotion
- Counselling Services
- International Centre

5 Marion McCain Building
- Security Services (Campus security, parking passes and lost property)

6 1321 Edward Street
- Black Student Advising and MultiFaith Centre

7 B Building
- Sexton Student Services and Student Support Centre
- Sexton Bookstore

HALIFAX

1. **Waterfront boardwalk** – 4 km of local sites, shops and restaurants.
2. **Neptune Theatre** – Take in a show from Atlantic Canada's largest professional theatre company.
3. **The Maritime Museum of the Atlantic** – Learn about the Titanic, the Halifax Explosion and other important Maritime history.
4. **Halifax Seaport Farmers' Market** – A must-see with vendors selling fresh local food and crafts.
5. **Pier 21** – Visit the Canadian Museum of Immigration.
6. **The Commons** – Playground, ballfields, skate park and The Emera Oval (free ice skating).
7. **Citadel Hill** – A National Historic Site.
8. **Scotiabank Centre** – Playground, ballfields, skate park and The Emera Oval (free ice skating).
9. **Argyle Street** – Full of restaurants and great for people-watching.
10. **Harbour Ferry** – An incredible city view and only $2.50 for a roundtrip.

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