MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

FINAL EXAM | **Mathematics 3202** | WINTER 2005

Marks

[12] 1. A particle starts at point $(3, 2, 1)$ and moves with velocity

$$\vec{v} = \begin{bmatrix} 0 \\ \sqrt{3} \\ 3t^2 \end{bmatrix}$$

[3] (a) Find the position vector $\vec{r}(t)$

[2] (b) Find the acceleration vector $\vec{a}(t)$

[2] (c) Find speed at $t = 1$

[3] (d) Find the curvature at $t = 1$

[1] (e) Find the radius of the osculating circle at $t = 1$.

[1] (f) Find the distance traveled from $t = 1$ to $t = 7$ (set up only, do not evaluate)

[10] 2. Let $f(x, y) = \sqrt{x^2 + 2y^2}$

[2] (a) Find the gradient of $f(x, y)$.

[3] (b) Find equation of a plane tangent to the surface $z = f(x, y)$ at point $(-1, 2, 3)$

[3] (c) Use Lagrange multiplier method to find the minimum of $f(x, y)$ subject to constraint $x + y = 3$, and the point where this minimum occurs.

[2] (d) Sketch the constraint $x + y = 3$ and the level curve of $f(x, y)$ which passes via the point found in (c).

[5] 3. Find volume bounded by the sphere $x^2+y^2+z^2 = 4$ from above, and by the cone $z = \sqrt{x^2 + y^2}$ from below in the **first octant**.

[9] 4. Evaluate the line integral $\oint_C xy^2\, dx + x^3\, dy$, where C is a trapezoid with vertices $(0,0)$, $(3,0)$, $(3,4)$, $(0,1)$

(a) directly

(b) using Green's theorem $\int_C \vec{F}\, d\vec{r} = \int\int_S (\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y})\, dA$

[5] 5. Use the Divergence theorem $\int\int_S \vec{F} d\vec{S} = \int\int\int_E \text{div}\vec{F} dV$ to evaluate the flux accross the surface of the tetrahedron bounded by planes $x = 0$, $y = 0$, $z = 0$ and $4x + 2y + z = 2$ of vector field $\vec{F} = (x, 2y, 3z)$.

[9] 6. Verify Stokes Theorem $\int_C \vec{F}\, d\vec{r} = \int\int_S \text{curl}\vec{F} d\vec{S}$ for vector field $\vec{F} = (y, z, x)$ and surface S which is a part of paraboloid $z = x^2 + y^2 - 16$ that lies below the xy-plane.