# MEMORIAL UNIVERSITY OF NEWFOUNDLAND

## DEPARTMENT OF MATHEMATICS AND STATISTICS

---

FINAL EXAM – ANSWERS **M-3202** FALL 2003

---

Marks

[25] 1. The velocity of a charged particle moving in a magnetic field and subject to the Lorentz force is the vector function

$$\mathbf{v}(t) = \langle \sin t,\ \cos t,\ 3 \rangle.$$

The initial position of the particle is $\mathbf{r}(0) = \langle 0, 0, -3 \rangle$.

[2] (a) Find the acceleration of the particle as a vector-function of $t$.

$$\text{Ans :} \qquad \langle \cos t,\ -\sin t,\ 0 \rangle$$

[5] (b) Find the position as a vector-function of $t$. Can you identify the shape of the trajectory?

$$\text{Ans :} \qquad \langle 1 - \cos t,\ \sin t,\ -3 + 3t \rangle$$

The trajectory is a helix.

[4] (c) Find the distance traveled from $t = 0$ until $t = 10$.

$$\text{Ans :} \qquad 10\sqrt{10}$$

[5] (d) Find an equation of the normal plane to the trajectory at the initial position.

$$\text{Ans :} \qquad y + 3z + 9 = 0$$

[9] (e) Find an equation of the osculating plane to the trajectory at the initial position.

$$\text{Ans :} \qquad 3y - z - 3 = 0$$

---

[10] 2. For the given function

$$F(x, y) = \frac{y^2}{2x + y^2}$$

[5] (a) Give a parametric description of a curve along which $\lim\limits_{(x,y)\to(0,0)} F(x, y) = -1$.

$$\text{Ans :} \qquad \text{For example} \quad \mathbf{r}(t) = \langle -t^2,\ t \rangle$$

[5] (b) Is the function $F(x, y)$ continuous at the point $(0, 0)$? Explain your answer.

**Ans:** It isn't. In (a) we found that the limit of $F(x(t), y(t))$ along a certain curve is $-1$. Now take another curve $\mathbf{r}(t) = \langle 0,\ t \rangle$; the limit along it is $1 \neq -1$.

---

[10] 3.

[5] (a) Find point(s) on the surface $z = y^2 - x^2 - 1$ where the normal line is parallel to the vector $\langle -1,\, 1,\, 1\rangle$.

$$\text{Ans :} \qquad \left(-\frac{1}{2},\, -\frac{1}{2},\, -1\right)$$

[5] (b) Find the equation of the tangent plane to the surface at the point where $x = 1$ , $y = 2$.

$$\text{Ans :} \qquad 2(x-1) - 4(y-2) + (z-2) = 0$$

[6] 4. Find the rate of change of the function $F(x, y) = \sinh(xy)$ at the point $(\ln 2,\, 1)$ in the direction $\theta = \pi/3$.

$$\text{Ans :} \qquad \frac{1+\sqrt{3}\ln 2}{2}\cosh \ln 2 \quad = \frac{5(1+\sqrt{3}\ln 2)}{8}$$

---

[14] 5. Consider the vector field $\vec{F}(x, y) = (2x - y,\; 2y - x)$.

[4] (a) Show that it is concervative.

$$\text{Ans :} \qquad \partial_x F_2 = -1 = \partial_y F_1$$

[5] (b) Find the potential function $f(x, y)$.

$$\text{Ans :} \qquad f(x, y) = x^2 - xy + y^2 + C$$

[5] (c) Evaluate (by any means) the line integral of $\vec{F}(x, y)$ along the segment of parabola $y = x^2$, $x \in [-1, 2]$.

Ans : 9

---

[8] 6. Evaluate by reversing the order of integration

$$\int_0^1 \int_x^1 e^{-y^2}\, dy\, dx.$$

$$\text{Ans :} \qquad \frac{1}{2}\left(1 - \frac{1}{e}\right)$$

[9] 7. Evaluate the line integral $\oint_C y dx - x dy$, where the curve $C$ is the circle $x^2 - 2x + y^2 = 3$ in two ways:

[5] (a) directly

[4] (b) using Green's theorem

Ans : $-\,8\pi$ in both cases

[6] 8. Find the volume of the solid tower with square base $0 \le x \le 1,\ \ 0 \le y \le 1$ at the level $z = 0$ and the roof described by the equation $z = \sqrt{xy}$.

$$\text{Ans :} \quad \frac{4}{9}$$

---

[12] 9. Solve **ONE** of the following problems. (If you solve more than one, you will receive the mark for the best out of 3 solutions, plus bonus 50% of the mark for the second best solution plus 25% of the mark for the third one.)

[12] (a) Use the Lagrange Multipliers Method to find maximum of the function $F(x, y) = x^2y$ with $x \ge 0, y \ge 0$, subject to the constraint $x + y = 1$.

$$\text{Ans :} \quad F\left(\frac{2}{3}, \frac{1}{3}\right) = \frac{4}{27}$$

[12] (b) Verify Stokes' theorem for the vector field $\vec{F} = \langle 3y, 2z, x\rangle$ and the surface $S$ which is the part of the paraboloid $z = 9 - x^2 - y^2$ that lies above the plane $z = 5$.

$$\text{Ans :} \quad -12\pi \quad \text{for both the LHS and the RHS}$$

[12] (c) Use the Divergence theorem to evaluate the flux across the surface of the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 2)$ of the vector field $\vec{F} = \langle xy, y^2, x^2\rangle$.

$$\text{Ans :} \quad \frac{1}{4}$$