Math 3202 **Answers** **Assignment #6**

1. Evaluate the line integral along given curve

   (a) $\int ye^x\,ds$, along the line segment jointing $(1,2)$ to $(4,7)$.
   Solution Line segment has equations $x = 1+3t$, $y = 2+5t$, $0 \le t \le 1$.
   Then $\int ye^x\,ds = \int_0^1 (2+5t)e^{1+3t}\sqrt{34}\,dt = \sqrt{34}(16e^4 - e)/9$.

   (b) $\int (xy + \ln x)\,dy$, along the arc of the parabola $y = x^2$ from $(1,1)$ to $(3,9)$.
   Solution Line segment has equations $x = t$, $y = t^2$, $1 \le t \le 3$.
   Then $\int (xy+\ln x)\,dy = \int_1^3 (t^3 + \ln t)\,2t\,dt = 464/5 + 9\ln 3$.

   (c) $\int x^2 z\,ds$, along the line segment jointing $(0,6,-1)$ to $(4,1,5)$.
   Solution Line segment has equations $x = 4t$, $y = 6-5t$, $z = -1+6t$ $0 \le t \le 1$. Then $\int x^2 z\,ds = \int_0^1 16t^2(-1+6t)\sqrt{77}\,dt = \sqrt{77}\,(56/3)$.

   (d) $\int (2x+9z)\,ds$, along the arc $x = t$, $y = t^2$, $z = t^3$, $0 \le t \le 1$.
   Answer: $(14^{3/2} - 1)/6$.

   (e) $\int z\,dx + x\,dy + y\,dz$, along the arc $x = t^2$, $y = t^3$, $z = t^3$, $0 \le t \le 1$.
   Answer: 3/2

2. Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = (yz, xz, xy)$ and $\vec{r} = (t, t^2, t^3)$, $t \in [0,1]$.
   Answer: 1.

3. Determine whether or not $\vec{F}$ is a concervative vector field. If it is, find its potential function $f(x,y)$.

   (a) $\vec{F} = (x^3 + 4xy, 4xy - y^3)$
   Answer: It is not conservative.

   (b) $\vec{F} = (e^y, xe^y)$
   Answer: It is conservative. The potential function is $xe^y + C$.

4. Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = (y, x+2y)$ along a smooth curve that starts at $(0,1)$ and ends at $(2,1)$. Hint: use the Fundamental Theorem of Calculus.
   Solution: The vector field is concervative. The potential is $g(x,y) = xy + y^2 + C$.
   Thus $\int_C \vec{F} \cdot d\vec{r} = g(2,1) - g(0,1) = 2$.