Math 3202 **Answers** **Assignment #4**

1. Find max and min values of the function $f(x, y)$ along the given curve. Plot the curve and the level curves of the function in the vicinity of the points where the function achieves max or min values.

   (a) $f(x, y) = 4x + 6y$, curve: $x^2 + y^2 = 13$.
   Answer. Solve $2\lambda x = 4$, $2\lambda y = 6$, $x^2 + y^2 = 13$ to get $\lambda = 1, x = 2, y = 3$ and $\lambda = -1, x = -2, y = -3$. Evaluate $f(2, 3) = 26$ and $f(-2, -3) = -26$. Thus maximum is achieved at point $(2, 3)$, and minimum at point $(-2, -3)$. At these points the level curves (lines) are tangent to the constraint (circle).

   (b) $f(x, y) = x^2 - y^2$, curve: $x^2 + y^2 = 1$.
   Answer Solve $2x = 2\lambda x$, $-2y = 2\lambda y$, $x^2 + y^2 = 1$ to get $x = \pm 1, y = 0$ and $x = 0, y = \pm 1$. Evaluate $f(\pm 1, 0) = 1$ and $f(0, \pm 1) = -1$. Thus maximum is achieved at points $(\pm 1, 0)$, and minimum at point $(0, \pm 1)$. At these points the level curves (hyperbolas) are tangent to the constraint (circle).

2. Evaluate the integral and sketch the domain of integration.

   (a) $\int_1^2 \int_y^2 xy\, dx\, dy$=9/8.

   (b) $\int_0^1 \int_0^x \sqrt{1 - x^2}\, dy\, dx = 1/3$.

   (c) $\int\int_D x \cos y\, dA = 0.5(1 - \cos 1)$, where region D is bounded by $y = 0$, $y = x^2$, $x = 1$.

   (d) $\int\int_D e^{x/y}\, dA = 0.5(e^4 - 4e)$,
   where region D is bounded by $1 \le y \le 2$, $y \le x \le y^3$.

   (e) $\int_0^1 \int_{3y}^3 e^{x^2}\, dx\, dy = (e^9 - 1)/6$.
   Hint: reverse the order.

   (f) $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3 + 1}\, dx\, dy = (2/9)(2^{3/2} - 1)$.

   (g) $\int\int_D \cos(x^2 + y^2)\, dA = (\pi/2) \sin 9$,
   where region D is above the x-axis and within $y^2 + x^2 = 9$.

   (h) $\int\int_D e^{-x^2 - y^2}\, dA = (\pi/2)(1 - e^{-4})$,
   where region D is bounded by y-axis and $x = \sqrt{4 - y^2}$.

3. Find the area of the surface:

   (a) the part of the surface $z = 1 + 3x + 2y^2$ that lies above the triangle with vertices $(0, 0)$, $(0, 1)$, $(2, 1)$.
   Answer: $(1/24)(26^{3/2} - 10^{3/2})$

   (b) the part of the elliptic paraboloid $z = 4 - x^2 - y^2$ that lies above the xy-plane.
   Answer: $(\pi/6)(17\sqrt{17} - 1)$

   (c) the part of the hyperbolic paraboloid $z = y^2 - x^2$ that lies between cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.
   Answer: $(\pi/6)(17\sqrt{17} - 5\sqrt{5})$

   (d) the part of the sphere of radius 4 and center at the origin, that lies abone the plane $z = 1$.
   Answer: $\int_0^{2\pi} \int_0^{\sqrt{15}} \sqrt{\frac{r2}{16 - r^2} + 1}\, r\, dr\, d\theta = 24\pi$

4. Evaluate

(a) $\int_0^1 \int_0^z \int_0^y ze^{-y^2}\, dx\, dy\, dz = (4e)^{-1}$.

(b) $\int\int\int_E ydV = 4/3$,
where E is bounded by planes $x = 0$, $y = 0$, $z = 0$ and $2x + 2y + z = 4$.

(c) $\int\int\int_F zdV = 27/8$,
where F is bounded by the cylinder $y^2 + z^2 = 9$ and the planes $x = 0$, $y = 3x$, $z = 0$ in the first octant.