Math 3202 **Due Tue March 7** **Assignment #5**

1. Evaluate the line integral of the vector field

   (a) $\int_C xy\,dx + (x-y)\,dy$, where C consists of line segments from (0,0) to (2,0) and from (2,0) to (3,2);

   (b) $\int_C \sin x\,dx + \cos y\,dy$, where C consists of the top half of the circle $x^2+y^2=1$ from (1,0) to (-1,0) and line segments from (-1,0) to (-2,3).

2. Evaluate $\int_C \vec{F}\cdot d\vec{r}$, where

   (a) $\vec{F} = (\,x^2y^3;\ -y\sqrt{x}\,)$, and $\vec{r} = (t^3; -t^3)$, $0 \le t \le 1$;

   (b) $\vec{F} = (\,x(x^2+y^2)^{-1/2};\ y(x^2+y^2)^{-1/2}\,)$, and $\vec{r} = (t, 1+t^2)$, $-1 \le t \le 1$.

3. Determine whether or not $\vec{F}$ is concervative vector field, and if yes, find the potential.

   (a) $\vec{F} = (2x\cos y - y\cos x; -x^2\sin y - \sin x)$;

   (b) $\vec{F} = (1 + 2xy + \ln x; x^2)$.

4. Show that the line integral is path-independent and evaluate the integral along any path from (1,0) to $(2, \pi/4)$

   (a) $\int e^y\,dx + xe^y\,dy$;

   (b) $\int \tan y\,dx + x\sec^2 y dy$.

5. Find the work done by force field $\vec{F} = (\,(y/x)^2;\ -2(y/x)\,)$ moving an object from (1,1) to (4,-2).

6. Consider vector field $\vec{F} = (\,-y(x^2+y^2)^{-1};\ x(x^2+y^2)^{-1}\,)$.

   a) Is $\vec{F}$ conservative?

   b) Calculate line integrals $\int_C \vec{F}\cdot d\vec{r}$ along lower and upper halfs of the circle $x^2+y^2=1$ from (1,0) to (-1,0). Is the line integral path-independent?

   c) Is there a contradiction? Explain.

7. Determine whether or not the given set is open, connected, simply-connected?

   a) $\{(x,y)|\ x<0, y>0\}$;

   b) $\{(x,y)|\ |x|>0\}$;

   c) $\{(x,y)|\ x^2+y^2 \le 5\}$;

   d) $\{(x,y)|\ x^2+y^2 > 5\}$.