Math 3202 **Answers** **Assignment #1**

1. For each vector-function $\vec{r}(t) = (x(t), y(t))^T$, $t \in [a, b]$ given below (a-d)

   1. sketch $x(t)$ and $y(t)$

   2. sketch the curve in xy-plane, showing all details such as slope, asymptotes, vertices etc. Name the curve.

   3. find the velocity vector $\vec{v}(t)$ and speed $v(t)$;

   3a. Plot vector $\vec{v}(0)$ for each curve on the same figure as the curve.

   4. find the acceleration vector $\vec{a}(t)$;

   5. set up the integral representing the length of the curve;

   6. find the curvature at $t = 0$

   (a) $x = 5\sin(2t)$, $y = 3\cos(2t)$, $0 \le t \le \pi/2$

   Answer:
   2. Right-half of the Ellipse $x^2/25 + y^2/9 = 1$ with vertices at $(0, \pm 3)$ $(5, 0)$
   3. $\vec{v} = (10\cos(2t), -6\sin(2t))$, $v(t) = \sqrt{64\cos^2(2t) + 36}$
   3a) $\vec{v}(0) = (10, 0)$.
   4. $\vec{a} = (-20\sin(2t), -12\cos(2t))$.
   5. $l = \int_0^{\pi/2} \sqrt{64\cos^2(2t) + 36}\, dt$
   6. $\kappa(0) = 3/25$.

   (b) $x = 5\tan(2t)$, $y = 3\sec(2t)$, $0 \le t \le \pi/8$

   Answer: 2. A segment of hyperbola $-x^2/25 + y^2/9 = 1$ from point $(0, 3)$ to point $(5, 3\sqrt{2})$
   3. $\vec{v} = (10\sec^2(2t), 6\sec(2t)\tan(2t))$, $v(t) = 2\sec(2t)\sqrt{25\sec^2(2t) + 9\tan^2(2t)}$
   3a) $\vec{v}(0) = (10, 0)$.
   4. $\vec{a} = (40\sec^2(2t)\tan(2t), 12\sec^3(2t) + 12\sec(2t)\tan^2(2t))$.
   5. $l = 2\int_0^{\pi/8} \sec(2t)\sqrt{25\sec^2(2t) + 9\tan^2(2t)}\, dt$
   6. $\kappa(0) = 3/25$.

   (c) $x = t^3$, $y = 1 - t^3$, $0 \le t \le 1$

   Answer:
   2. A segment of line $y = 1 - x$ from point $(0, 1)$ to $(1, 0)$.
   3. $\vec{v} = (3t^2, -3t^2)$, $v(t) = 3\sqrt{2}t^2$
   3a) $\vec{v}(0) = (0, 0)$.
   4. $\vec{a} = (6t, -6t)$.
   5. $l = 3\sqrt{2}\int_0^1 t^2\, dt = \sqrt{2}$
   6. $\kappa(0) = 0$.

(d) $x = 5t^9 + 1$, $y = 3t^3$, $0 \le t \le 1$

Answer:

2. A segment of cubic parabola $x = (5/9)y^3 + 1$ from point $(1,0)$ to $(6,3)$.
3. $\vec{v} = (45t^8, 9t^2)$, $v(t) = 9t^2\sqrt{25t^{12}+1}$

3a) $\vec{v}(0) = (0,0)$.

4. $\vec{a} = (360t^7, 18t)$.
5. $l = 3\sqrt{2}\int_0^1 t^2\,dt = \sqrt{2}$
6. $\kappa(t) = 2430t^3/(729(25t^{12}+1)^{3/2})$, thus $\kappa(0) = 0$.

2. Find parametric equations for the sides of the trapezoid with vertices at points $A(-1,0)$, $B(-1,1)$, $C(1,1)$, $D(2,0)$.

Answer:

$A \to B$ $x = -1$, $y = t$, $t \in [0,1]$

$B \to C$ $x = 2t - 1$, $y = 1$, $t \in [0,1]$

$C \to D$ $x = 1 + t$, $y = 1 - t$, $t \in [0,1]$

$D \to A$ $x = 2 - 3t$, $y = 0$, $t \in [0,1]$

3. Find velocity vector $\vec{v}(t)$ if

(a) the acceleration vector is
$\vec{a}(t) = \dfrac{2t}{1+t^2}\mathbf{i} + \ln t\,\mathbf{j} + e^{-4t}\,\mathbf{k}$, $t > 0$

Answer:

$$\vec{v}(t) = \left(\ln(1+t^2) + c_1;\ t\ln t - t + c_2;\ -\frac{e^{-4t}}{4} + c_3\right),$$

where c_1, c_2, c_3 are arbitrary constants.

(b) the position vector is
$\vec{r}(t) = \dfrac{2t}{1+t^2}\mathbf{i} + \ln t\,\mathbf{j} + e^{-4t}\,\mathbf{k}$, $t > 0$

Answer:

$$\vec{v}(t) = \left(\frac{2(1-t^2)}{(1+t^2)^2};\ \frac{1}{t};\ -4e^{-4t}\right).$$