Mathematics 3205

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Grade Level: Grade 12

Time Allowed: 1 hour

Number of Students: 29 (maximum)

Goal (KSCO):

- Students will be expected to develop an understanding of factorial notation and apply it to calculating permutations
- Students will be expected to develop and apply formulae to evaluate permutations

Objectives:

- Give students examples of how to arrange objects and understand the meaning of permutations
- Make the students familiar with factorial notation and operations with factorials
- Use examples to help students derive a formula for permutations
- Make students aware of technological aids in solving permutations
- Incorporate a rich assessment task that allows students to explore a more complex problem

Materials:

- Whiteboard markers and a whiteboard eraser
- Sets of six differently coloured blocks (each group of three or four will need one set)
- Graphing calculators
- Laptop computer
- Assignments

Motivation

- You are a prisoner and you have to hammer out licence plates with certain specifications. The licence plates consist of 3 different letters and 3 different digits. The first letter must be a vowel, while the second two letters must be consonants. The first digit must be even, while the other two digits must be odd. How many licence plates will you have to make?

Lesson Procedure:

1. We will begin the class by asking students, as they enter, to write their names on the board. This will be used in an upcoming example.
2. We will introduce a complex example involving permutations as described above under "Motivation". We will not explore the solution of this problem; rather we will simply discuss with the class different ways to approach such a problem.
3. We will ask for four volunteers to engage in an activity involving a committee consisting of a president, a vice president, a secretary, and a treasurer. The goal is to discover how many different committees are possible, with the solution of 4 x 3 x 2 x 1 = 24.
4. We will use the above activity to define what is a permutation, and we will give a couple of simple examples.
   a. How many ways can you arrange the digits in the number 12578?
   b. How many ways could you all have walked into the classroom? (For this activity we will use their names that they wrote on the board earlier).
5. The previous activity will lead into the notion of factorials. We will define factorial and give a couple of simple examples. We will also define 0! = 1
   a. 5! (5! = 5 x 4 x 3 x 2 x 1 = 120)
   b. Given that 8! = 40320, what is 9!? (9! = 9 x 8! = 362880)
6. We will then explain the notion of cancelling factors, with another couple of examples.
   a. 6! / 4! (6 x 5 x 4!) / 4! = 6 x 5 = 30
   b. 15! / 12! (15 x 14 x 13 x 12!) / 12! = 15 x 14 x 13
7. Our next activity will involve the coloured blocks found in the classroom. We will divide the class into groups of 3 or 4, and each group will get a set of 6 differently coloured blocks. They will be asked to find the number 4 block "strings" that can formed, emphasizing that the order the colours matters. We will discuss with the class how they arrived at their answers, with the goal of arriving at the solution of 6 x 5 x 4 x 3 = 360 which can be rewritten in terms of factorials as 6! / 2!
8. Using the above example, we will derive the formula for permutations. Namely, if you are arranging r items from a group of n, which can be written as ${}_nP_r$ you will have n! / (n-r)! arrangements. We will use this formula in a couple of simple examples.
   a. How many ways can you choose six different doughnuts if you have 10 different types? Order of choice matters?
9. We will use a computer program to demonstrate the use of graphing calculators in solving permutations. We will check the solutions to the previous two examples using this method.
10. Now that the class has an understanding of permutations and how to solve them, we will go back to the original licence plate problem. We will first introduce a simpler example, asking them to find the number of licence plates that have three letters and three numbers. (26^3 x 10^3). We will then ask them to find the number of plates with three different letters and three different digits (26 x 25 x 24 x 10 x 9 x 8). We will then ask them to find the

number of licence plates that begin with the letter B and end with the number 2 where letters and digits cannot be repeated. ($_{25}P_2$ x $_{9}P_2$) Finally we will give them a few minutes to solve the original problem (5 x $_{25}P_2$ x 5 x $_{9}P_2$).

Closure:

Students should have gained an understanding of what is a permutation, how the formula was derived, and a few examples of where permutations can be used. We will discuss with the class what their thoughts are on permutations and if they can think of any real life situations where they may be useful.

Assessment:

Students will be given an assignment testing all aspects of what was taught during the class.

UNIT PLAN: UNIT 5 PROBABILITY

Unit Goals & Objectives:

After completing this course, students will be able to:

- Develop and apply simulations to solve counting problems
- Demonstrate an understanding that determining probability requires the quantifying of outcomes
- Demonstrate an understanding of the Fundamental Counting Principle and apply it to calculate probabilities of dependent and independent events
- Apply area diagrams and tree diagrams to interpret and determine probabilities of dependent and independent events
- Determine conditional probability
- Demonstrate an understanding of factorial notation and apply it to calculating permutations and combinations
- Distinguish between situations that involve permutations and combinations
- Develop and apply formulae to evaluate permutations and combinations
- Determine probabilities using permutations and combinations
- Demonstrate an understanding of binomial expansion and its connection to combinations
- Connect Pascal's Triangle with combinatorial coefficients
- Connect binomial expansions, combinations, and the probability of binomial traits
- Demonstrate an understanding of and solve problems using random variables and binomial distributions

Assumed Prior Knowledge:

Prior to beginning Unit 5 in Math 3205, students are assumed to understand:

- The concept of probability, including both theoretical and experimental probability and the differences between them
- The concepts of trials and simulations
- The concept of a Venn diagram, and how to use it
- The concept of dependent and independent events, and how to differentiate between them
- The concept of binomial expressions, and how to work with them
- The knowledge of how to use a TI-83 (or equivalent) graphics calculator

Learning Resources:

The principal resource used for this unit will be the student textbook, Mathematical Modelling Book 3, published by Nelson. Since this topic is a fundamental one in discrete mathematics, there are a plethora of other resources (both textbooks and websites) that may be drawn upon for problem design and alternate teaching methods—however, Mathematical Modeling will be the principal resource.

Timeline:

The teacher's resource of Mathematical Modeling 3 recommends 20 hours to teach Unit 5, and this is reflected by the curriculum document for Math 3205, which recommends 10-15% of the total course time and worth be devoted to Unit 5. Unit 5 is broken into 6 subsections, listed here with approximate timelines for them:

1. Probability & Quantifying Outcomes: 3 classes
2. Counting Problems & Probability: 6 classes
3. Combinations & Permutations: 4 classes
4. Counting Probabilities Using Combinations & Permutations: 2 classes
5. Applying Probability & Combinations To The Binomial Expansion: 2 classes
6. Binomial Probabilities: 3 classes

Students will be given a quiz after the first 9 classes (first 2 subtopics) to assess their understanding of the basic material before they are taught about combinations & permutations. Students will also have an assignment to be handed in & graded prior to the quiz (by class 8 or 9), as well as another assignment on the whole unit, to be handed in & graded prior to the unit test (by class 19 or 20). There will also be a unit assignment, and a unit test, which cover the whole unit. So, allowing one class for the quiz, one class for handing back the quiz & reviewing problem areas, one class for test review/general questions, and one class for the test itself, this unit will take 24 classes (or approximately 5 weeks with daily classes) to teach.

Tools:

For this unit, students will make use of their TI-83 (or equivalent) calculators for most of the calculations to be done in the unit, and to check their work. Using a calculator-to-USB cable link, the teacher can install the Probability Simulation Application (http://education.ti.com/educationportal/sites/US/productDetail/us_prob_sim_83_84.html) on the students' calculators that will run simulations on dice, cards, random numbers, etc. The calculator will also be used to aid in complex factorial computations that the students will carry out in problem solving.

The top five outcomes for *Number Sense and Counting Techniques* are:

1) **Students will be able to develop a meaning for integers and represent and compare quantities with them. They will be able to compare and contrast the properties of numbers and number systems, including the rational and real numbers. (NCTM Principles and Standards):**
Numbers are classified according to type with the first type being the counting or natural numbers followed by whole numbers, integers, rationals and irrationals. It is essential for students to know what the terms mean when they hear them. For example, when students hear their teacher talk about integers they need to understand that the term refers to the counting numbers, their negatives and zero. Each type of number has its own properties which will help them solve problems in algebra. As a result, students will develop an intuitive feel for numbers and their relationships which will form when students solve problems for themselves.

2) **Students will be able to use the associative and commutative properties of addition and multiplication. They will also learn the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals (NCTM Principles and Standards):**
There are many things in mathematics that can be added and multiplied such as numbers, vectors, matrices, functions, equations and sets. Students should understand as an abstract (?) operation that multiplication is the same as addition. Both are binary operations that satisfy the same set of axioms such as the associative and commutative* operations. *not for matrices or cross product of vectors Students will understand that x(a+b) = xa + xb thus it will become clear that multiplication is more powerful than addition because you have to distribute the number you are multiplying to all numbers that are being added in the parenthesis. As a result of this outcome students will require prior knowledge of the order of operations. They will need to know that you perform the operations in order which can be represented as BEDMAS: Brackets, Parenthesis, Exponents, Multiplication or Division then Addition or Subtraction.

3) **Students will explore, recognize and represent and apply patterns and relationships both informally and formally (Foundation for Atlantic Canada Math Curriculum):**
Students at any level of schooling need opportunities to connect patterns to number ideas. Teachers can invite students to use blocks when examining a pattern to help make informal decisions and connections. Students can formally explore patterns by using a calculator to help them solve the problem mathematically involving a formula.

4) **Students should be able to develop a fluency in operations with real numbers, fractions, decimals, vectors and matrices using mental computation or paper and pencil calculations for simple cases and technology for more complicated cases (NCTM Principles and Standards):**
It is important for students to be able to select appropriate methods and tools for computing with real numbers, fractions, decimals, vectors and matrices. Mental computation, estimate, calculators or computers and paper and pencil are all very useful depending on the situation. Technology is a great tool that can influence the mathematics that is being taught and helps to enhance the students' learning. Technology can increase students' focus on more important

mathematics and can reduce the effort devoted to tedious computations. Technology can also represent math in ways that help students understand the concepts because they can confirm their answer when working on complex examples.

5) **Students will develop an understanding of factorial notation and apply it to calculating permutations (Department of Education of NL Curriculum Outcomes):**
It is important for students to understand the factorial notation because we need a simple way of writing the product of all positive whole numbers up to a given number. This idea of factorial notation guides the student to understand the formula for permutation. The student must first know the definition of a factorial before they can proceed into permutations. They can recall from the factorial section that n factorial (n!) is defined as n! = n x (n-1) x (n-2) …3 x 2 x 1. As a result, this leads students to recognize that the permutation theorem uses factorials. Students should understand how they can apply permutations to real life situations because this can easily engage them in thinking about time when they can use permutations in their daily routine. This can allow students to interact with their own learning since they can ask themselves about a daily activity where order matters such as brushing their teeth, getting dressed or baking a cake.

Numbers and Counting: Facts, Statements, Producers and Formula's

Types of Numbers:

- *Real Numbers*: the set of all numbers.
- *Whole Numbers*: the set of all positive integers, including 0.
- *Natural Numbers*: the set of all positive integers, **not** including 0.
- *Integers*: the set of all positive and negative whole numbers, including 0.
- *Rational Numbers*: the set of all number that can be expressed as a fraction. Such that the numerator and denominator are integers and the **denominator cannot be 0**. Rational numbers has a repeating or terminating decimal.
- *Irrational Numbers*: the set of all numbers that cannot be expressed as a fraction. Where the decimals are non-repeating or non-terminating.
- *Complex Numbers*: the set of numbers that take the form $\beta + \delta i$, where β and δ are real numbers and i is the imaginary number equal to $\sqrt{-1}$.

Place Value:

Thousands	Hundreds	Tenths	Ones	Decimal	Tenths	Hundreds	Thousands
1000	100	10	1	•	0.1	0.01	0.001

Fractions:

For all $\alpha, \beta, \gamma, \delta \in Reals, such\ that\ \gamma, \delta \neq 0,$

1. $\frac{\alpha}{\beta} + \frac{\gamma}{\delta} = \frac{\alpha\delta+\gamma\beta}{\beta\delta}$

2. $\frac{\alpha}{\beta} - \frac{\gamma}{\delta} = \frac{\alpha\delta-\gamma\beta}{\beta\delta}$

3. $\frac{\alpha}{\beta} \times \frac{\gamma}{\delta} = \frac{\alpha\gamma}{\beta\delta}$

4. $\frac{\alpha}{\beta} \div \frac{\gamma}{\delta} = \frac{\alpha}{\beta} \times \frac{\delta}{\gamma} = \frac{\alpha\delta}{\beta\gamma}$

Computations:
For all $\alpha, \beta, \gamma \in Reals$,

1. Commutative: $\alpha + \beta = \beta + \alpha, \quad \alpha \times \beta = \beta \times \alpha$
2. Associative: $\alpha + (\beta + \gamma) = (\alpha + \beta) + \gamma, \quad \alpha \times (\beta \times \gamma) = (\alpha \times \beta) \times \gamma$
3. Distributive: $\alpha \times (\beta + \gamma) = \alpha \times \beta + \alpha \times \gamma$
4. Identity: $\alpha + 0 = \alpha, \quad \alpha - 0 = \alpha, \ \alpha \times 1 = \alpha, \ \alpha \div 1 = \alpha$
5. Inverse: $\alpha + (-\alpha) = 0, \ \alpha \times \frac{1}{\alpha} = 1$

Prime and Composite Numbers:

- Prime numbers: are whole numbers that have only two factors, itself and one. ie, 7
- Composite Numbers: are whole numbers who have more than just itself and one. ie, 16

Prime decomposition of every number.

Exponential Thinking:
For all $\alpha, \beta \in Reals$, and $n, m \in$ integers,

- $\alpha^n \times \alpha^m = \alpha^{n+m}$
- $\frac{\alpha^n}{\alpha^m} = \alpha^{n-m}$
- $(\alpha^m)^n = \alpha^{mn}$
- $\alpha^{\frac{n}{m}} = \sqrt[m]{\alpha^n} \longleftarrow$ domain: possible values for a
- $(\alpha\beta)^n = \alpha^n \beta^n$

Order of Operations (BEDMAS):

1. Brackets
2. Exponents
3. Division
4. Multiplication
5. Addition
6. Subtraction

Foil: First, Outside, Inside, Last

$$(\alpha+\beta)(\gamma+\delta) = \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta$$

Basically same as the distributive law.

Percentage:

- $\mu\,\% = \frac{\mu}{100}$

"Students learn best when ideas are approached in a variety of ways"

- Abs value of a number $|a \cdot b| = |a| \cdot |b|$
- Basic counting rules (addition and multiplication) leading to permutations and combinations

Math 3205: Take home assignment

Name:

Answer all the questions below. Make sure to show all workings where possible.

1.)

(a) Compute 5! (2 marks)

(b) From part (a) Compute 6! (1 mark)

2.) Using technology compute 9! (1 mark)

3.) Compute $\frac{6!}{4!}$ (3 marks)

4.) Compute $\frac{n!}{(n-1)!}$ (3 marks)

5.) Find how many ways we can arrange two letter words out of the word MATH.

(a) Write down all possible arrangements (2 marks)

(b) By using the permutation equation (2 marks)

(c) Check answer by using technology (1 mark)

6.) You are given six different colour stars; Red, Orange, Blue, Brown, Green and Yellow. How many different ways can you arrange 4 of these stars.

(a) Solve by general explanation (3 marks)

(b) Solve by using the permutation question (3 marks)

(c) Check your answer by using technology (1 mark)

7.) Find how many ways we can arrange three letter words out of the name HILARY, given that the first letter has to be an H. (4 marks)

Solutions:

1.) (a) $5! = 5 \times 4 \times 3 \times 2 \times 1 = 20 \times 3 \times 2 \times 1 = 60 \times 2 \times 1 = 120 \times 1 = 120$

(b) From part (a) we know that $5! = 120$ therefore,

$$6! = 6 \times 5! = 6 \times 120 = 720$$

2.) From using the calculator, we know that $9! = 362880$

3.) $\frac{6!}{4!} = \frac{6\times5\times4\times3\times2\times1}{4!} = \frac{6\times5\times4!}{4!} = 30$

4.) $\frac{n!}{(n-1)!} = \frac{n\times(n-1)\times(n-2)\times\cdots(3)(2)(1)}{(n-1)\times(n-2)\times\cdots(3)(2)(1)} = n$

5.) (a) 12 ways

MA	AM	TM	HM
MT	AT	TA	HA
MH	AH	TH	HT

(b) $nPr = \frac{n!}{(n-r)!} = \frac{4!}{(4-2)!} = \frac{4\times3\times2\times1}{2\times1} = 4 \times 3 = 12$

(c) By using your calculator, you get 12.

6.)

(a) Red, Orange, Blue, Brown, Green and Yellow

For our first position we have 6 different stars to choose from

Since we just took one star, we are now left with 5 different color stars, hence, 5 choices

Since we just took two stars, we are now left with 4 different color stars, hence, 4 choices

For our last position, we have taken 3 stars therefore 3 are left, hence, 3 choices.

$$6 \times 5 \times 4 \times 3 = 360$$

(b) $nPr = \frac{n!}{(n-r)!} = \frac{6!}{(6-4)!} = \frac{6\times5\times4\times3\times2\times1}{2\times1} = 6 \times 5 \times 4 \times 3 = 360$

(c) from using your calculator, you get 360.

7.) 20 different ways.

1	5	4