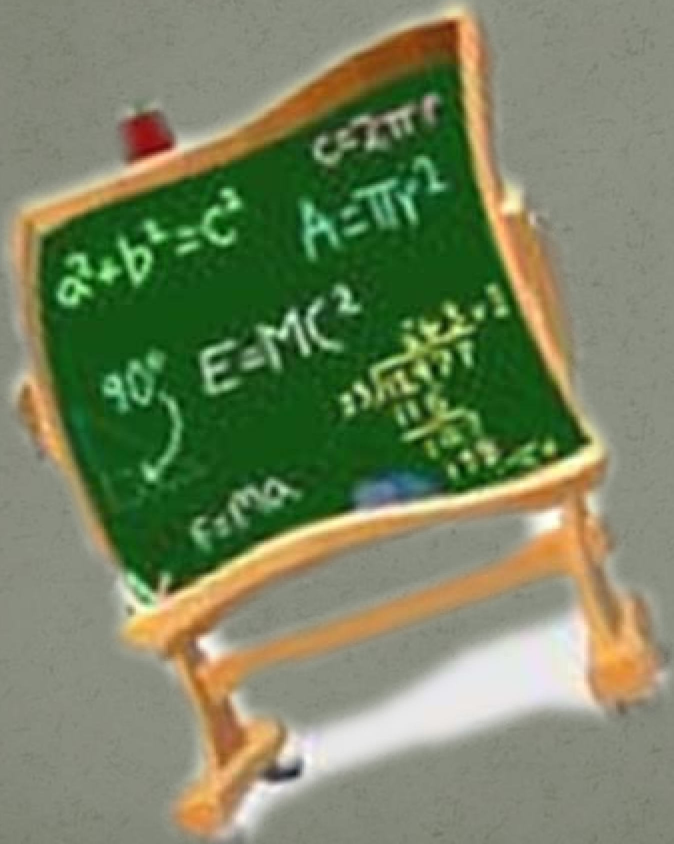
Developing Mathematical Connections
Chapter 3



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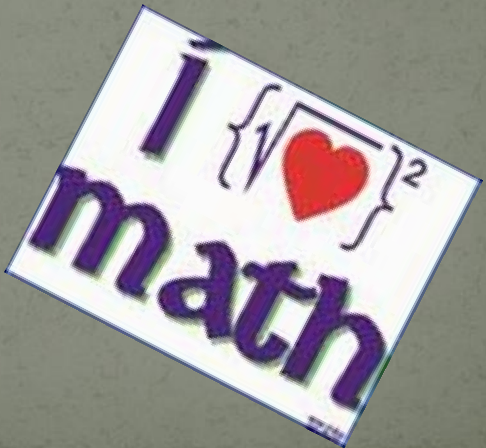
Introduction

- Understand world from mathematical perspective
- Build cohesive & comprehensive picture of math
  - Need to make connections
- Help form and strengthen these connections



Introduction (Cont'd)

Connections Across:
- Middle years
- Curriculum areas
- Lessons

- Connections within:
  - Lessons

- Connections Through:
  - Applications
  - Modeling

- Conclusions

Middle Years

- Transition from primary to secondary
- Middle Years Numeracy Research (MYNR) Project
  - Significant drop in performance (1999)
- Actively engage students
  - Unique teaching strategies
  - Focus on connections & meaning
- Activities appropriate to learner's needs/interests

Middle Years (Cont'd)

- MYNR Research
  - Developed emergent numeracy profile

- Discriminating factors of performance
  - Rational numbers
  - Metacognitive activity
  - Dealing with patterns

- Teachers can draw on materials

Curriculum Area

- Cross curricular cooperation
  - Using math in other subject areas
- Works better in certain subjects
  - Emanates from math classroom
  - No formal cooperation
  - Allocation of curriculum & territory
- Formal cooperation
  - Payoff between time & learning
  - Math must be anchor point

Lessons

- Deliberately structure the making of connections
  - Students understand coherency across topics
- Suitable content of generative problems
  - Real-world situation
  - Mathematical context
- Focusing students on ideas & relationships

Making Connections Across Lessons

Goals

- For students to think independently.
- For students to develop a knowledge of interrelatedness of mathematical concepts.

Methods

- Triadic Dialogue
- Funneling
- Focusing

Triadic Dialogue

- Initiation- Response- Feedback.
- Teacher dominated communication.
- Teacher asks closed questions.
- Students recall facts or unique response expected by teacher.

Funneling

- Teacher controlled but not restricted.
- “Scaffolding”
- Teacher funnels students through a predetermined framework to a desired result.
- Students believe they are making connection for themselves.

Focusing

- Teacher asks probing questions.
- Students control/ teacher listens and guides.
- Problems are resolved collectively.
- Provides the teacher with feedback on students thoughts and understandings.

What does this mean in the Classroom?

Lecture only — High Teacher Control, Low Student Engagement

Triadic dialogue

Funneling

Focusing — High Student Engagement, Low Teacher Control

What can Students Gain?

- Have to learn to articulate their ideas
- Become comfortable with taking risks
- Develop connection making and critical thinking skills.
- Improved learning and understanding of subject area

What is Required of Teachers?

- Incorporate time for discussion
- Create a safe environment for risk taking.
- Are knowledgeable in subject area and pedagogy.
- Are confident in their ability to control the classroom
- Possess good listening and questioning skills

Connections Through Mathematical Applications

Real-World Application Problems

- Curriculum advocates making real world connections
  - Motivates and engages students
  - Ability to describe and analyze real-world situations

- Galbraith(1987)
  - Important but limited in function
  - 'Requires translation, interpretation and successful use of relevant mathematics'
  - Situation carefully described and relevant data given
    - Assumptions need to define outcome are explicitly provided

Purpose & approach: Application Problems

- Mathematical awareness and understanding
  - Fostered by active engagement
    - Finding & applying solutions to real-life problems
    - Problems within student's personal understanding
- Knowledge of students' interests is crucial
- Students could pose own problems

Approach & Levels of Application Problems

- Teacher could choose open tasks
  - Students have freedom to choose pathway
  - Pose questions of personal interest to solve

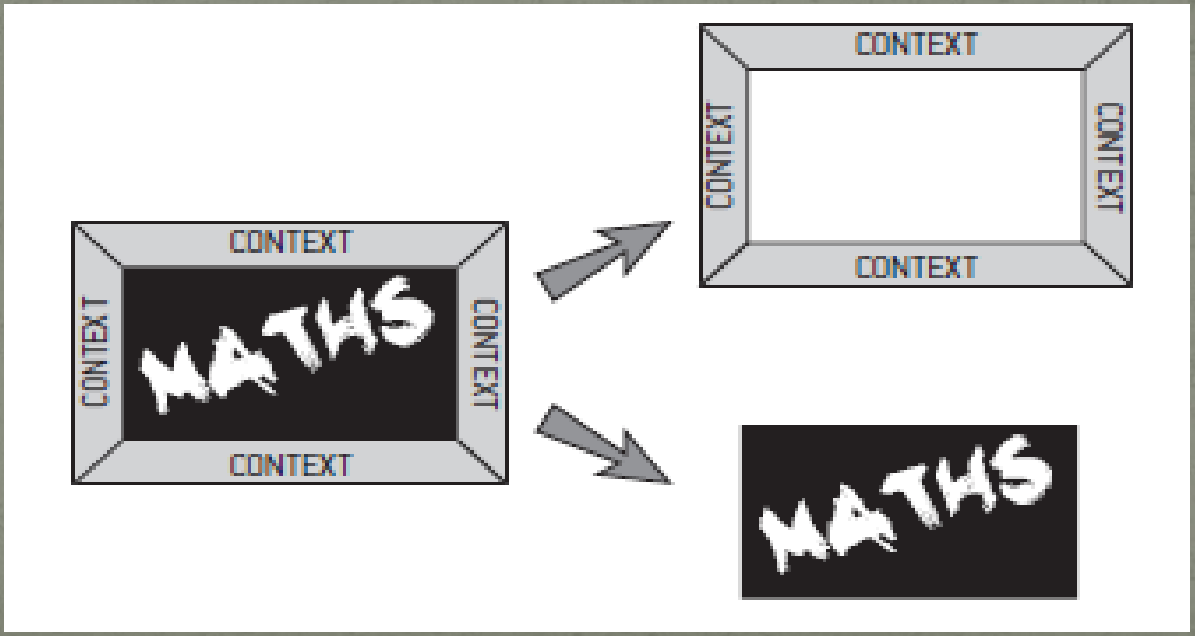
- Application problems most often used in class
  - More reduced task contexts than life-like ones

- 3 levels of embedding context to characterize problems
  - **Border problems**
  - **Wrapper problems**
  - **Tapestry problems**

Border Problems

- **Context** is merely **border** around **mathematics**

- Mathematics and context entirely separate
  - Can separate the two
  - Context does not obscure the mathematics



Border Problems

- Can be deceptively difficult for students
  - Particularly for lower secondary years

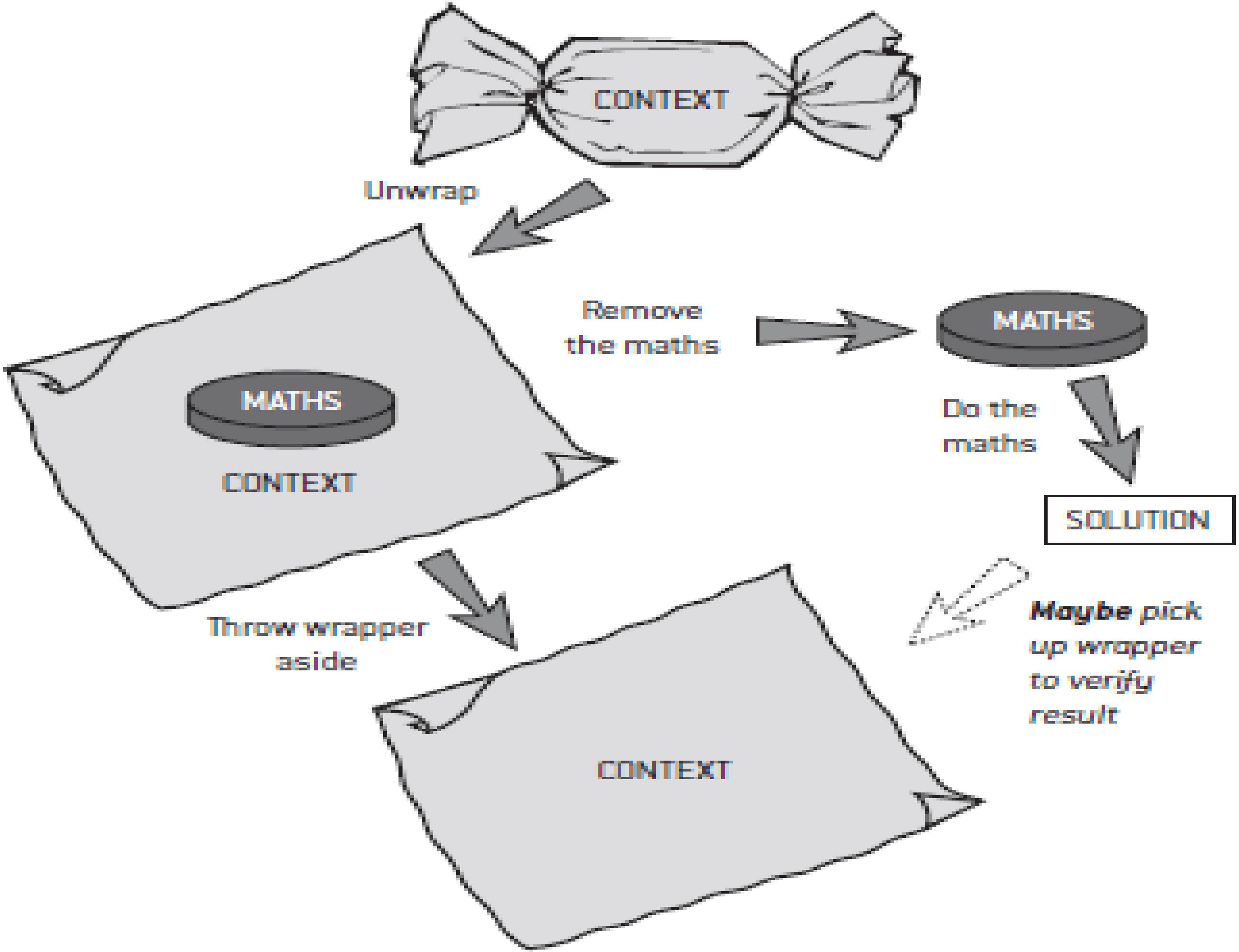
Microwave ovens

The number of radioactive emissions from a certain faulty microwave oven is given by $N_1 = 64(0.5)^t$ at t years from the time of use. The number of emissions from a second faulty microwave is given by $N_2 = 4^{15}(0.0625)^t$ at t years from the first time of use. Find out when both microwaves will emit the same number of radioactive emissions.

Source: Stillman (2002).

Wrapper Problems

- Mathematics are hidden within the context
  - Can be separated by unwrapping the mathematics
- Context can be thrown away
  - Only mathematics needed to solve problem
- Can pick up wrapper (context) once solved
  - Check to see solution makes sense
- Presence of context can make unwrapping challenging


CONTEXT
Unwrap
Remove
the maths
MATHS
MATHS
CONTEXT
Do the
maths
SOLUTION
Throw wrapper
aside
Maybe pick
up wrapper
to verify
result
CONTEXT

Wrapper Example

Road construction

A new traffic lane (minimum width 6 metres) is to be added to a section of highway which passes through a cutting. To construct the new lane, engineers need to excavate an existing earth bank at the side of the roadway, which is inclined at 25° to the horizontal. This will make the inclination steeper. Local council regulations will not allow slopes greater than 40° due to the potential for erosion. Decide whether the new traffic lane can be excavated without expensive resumption of properties at the top of the bank, which is 7 metres above the road surface.

Source: Galbraith & Stillman (2001, p. 306).

Tapestry Problems

- Context and mathematics are entwined/intermingled
  - Always sense that two are very much interrelated
- Continually refer back to context
  - Check that you are on right track
- Upper bound of these applications are true modelling problems

Tapestry Example

Drying out

There are many lakes in Australia that are dry for most of the time, only filling for short periods immediately after rain. Lake Eyre in South Australia is an example of one of these normally dry lakes. When a dry lake bed is filled with water, how rapidly will the lake empty?

Source: Henry & McAuliffe (1994, pp. 41–48).

Summary Review

- Over-use of border and wrapper problems leads to limited view of mathematical applications
  - Does not foster modes of thought associated with modelling of real-world situations

- Application problems provide useful bridge b/w contextualised practice problems of the past and full-blown modelling tasks

Connections Through Mathematical Modelling

What is modelling… Exactly?

- A math concept that happens to be relevant to a real-world problem…

What is modelling… Exactly?

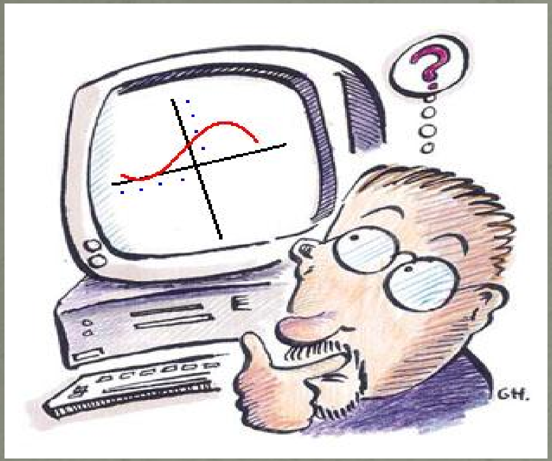
- …or a real-world problem that happens to be solvable with math?

  - CONTEXT of the problem
  - Problem-solving PROCESS
  - Acquisition of METAKNOWLEDGE

Why change the meaning now?

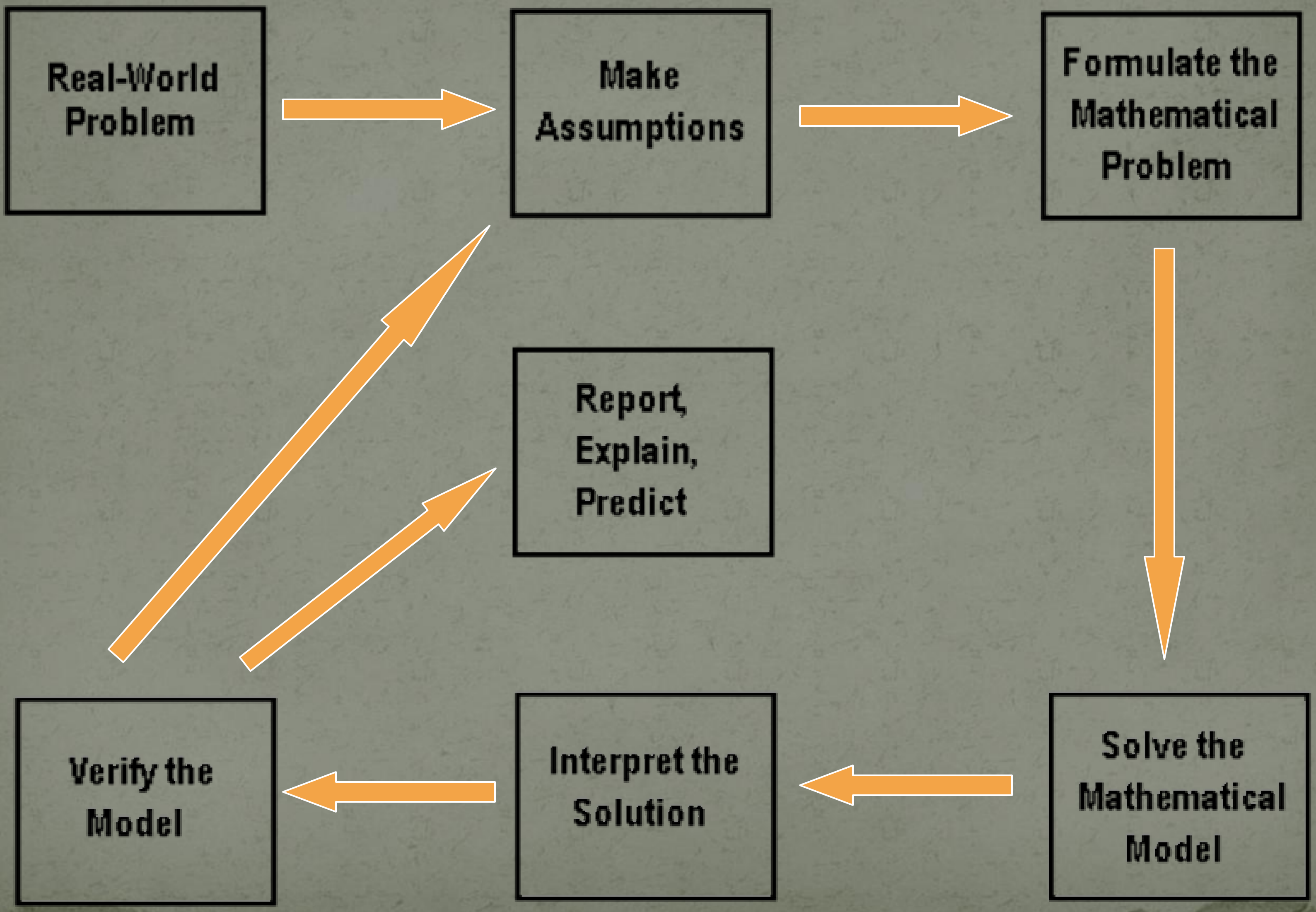
Why change the meaning now?

- Technology



- Student demands/expectations

The Modelling Process



- All decisions are made by the student.

- The student has the ability to change and adapt the model.

- The student is aware of the model’s limitations

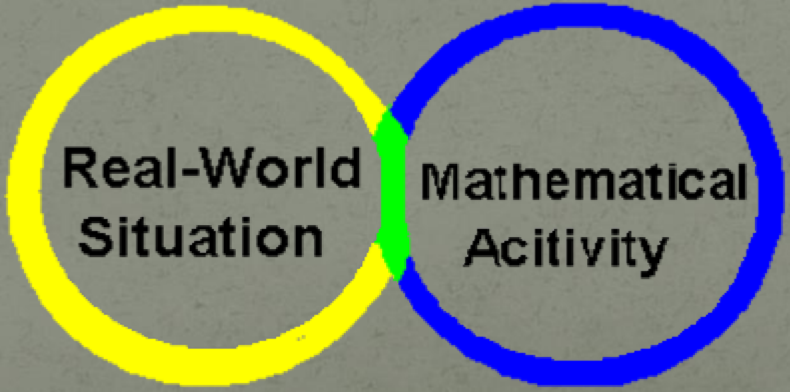
∴ Modelling > Application Problems

You decide-

What is the best way to represent…

- hitting a baseball?
- driving through the mountains?
- an eclipse of the moon?
- shooting a bird out of the sky?

Modelers must always be aware of the connection between the situation they are dealing with and the mathematics they are performing.



VALIDATION

II

REGULATORY MECHANISM

Modeling Sub-skills

- Formulating math questions
- Specifying assumptions
- Identifying variables
- Modelling objects & situations
- Generating & selecting relationships
- Estimating
- Validating & interpreting results

Sub Sub-skills Examples

- Types of assumptions:
  - Mathematical concepts

    -a shot put throw can be treated as a free projectile
  - Mathematical detail

    -the range of a projectile varies only slowly with the angle of projection
  - Modelling context

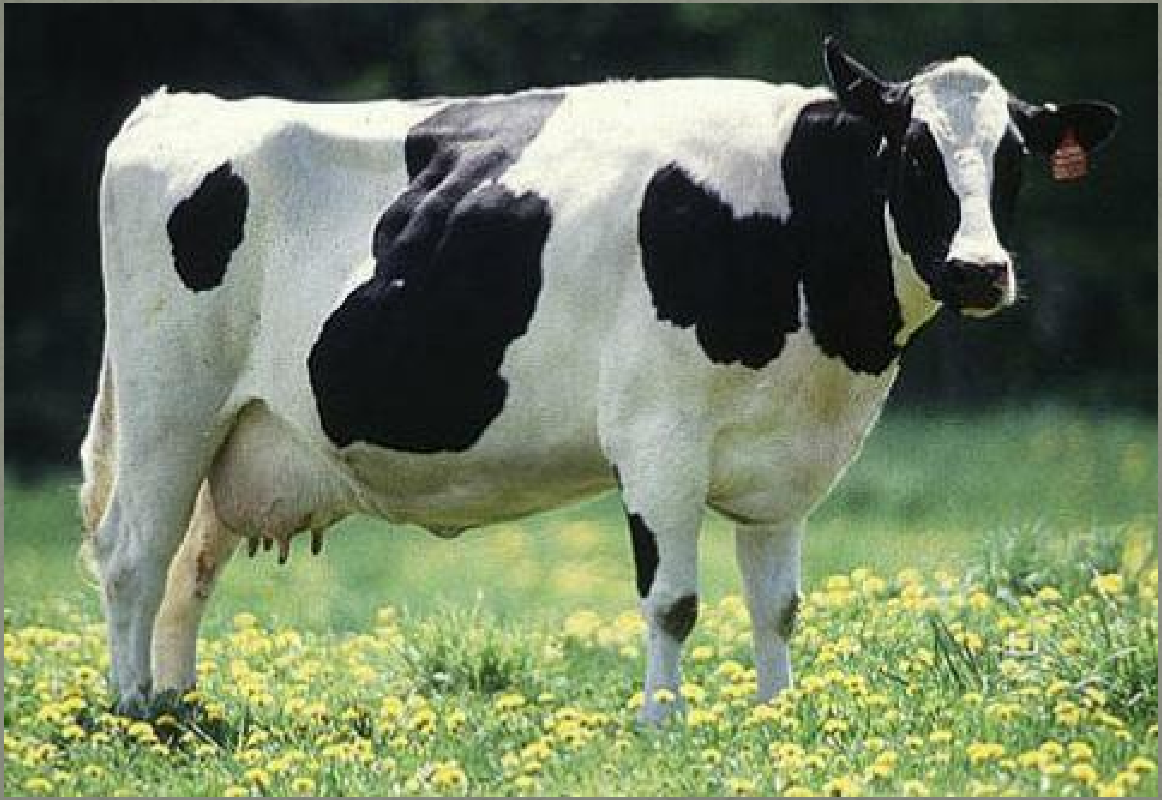
    -172 centimetres is a representative height for a female athlete
  - Solution process

    -10% is a reasonable estimate to compensate for air resistance

Example Modeling Problem

- CJD first classified in 1920s
- vCJD in 1996
- vCJD caused by same agent as Mad Cow Disease
- Prions create more prions
- Height of Mad Cow Disease in mid 1980s
- First case of vCJD in mid 1990s
- Affected persons live ~2 years
- 156 deaths from vCJD by August 4, 2006

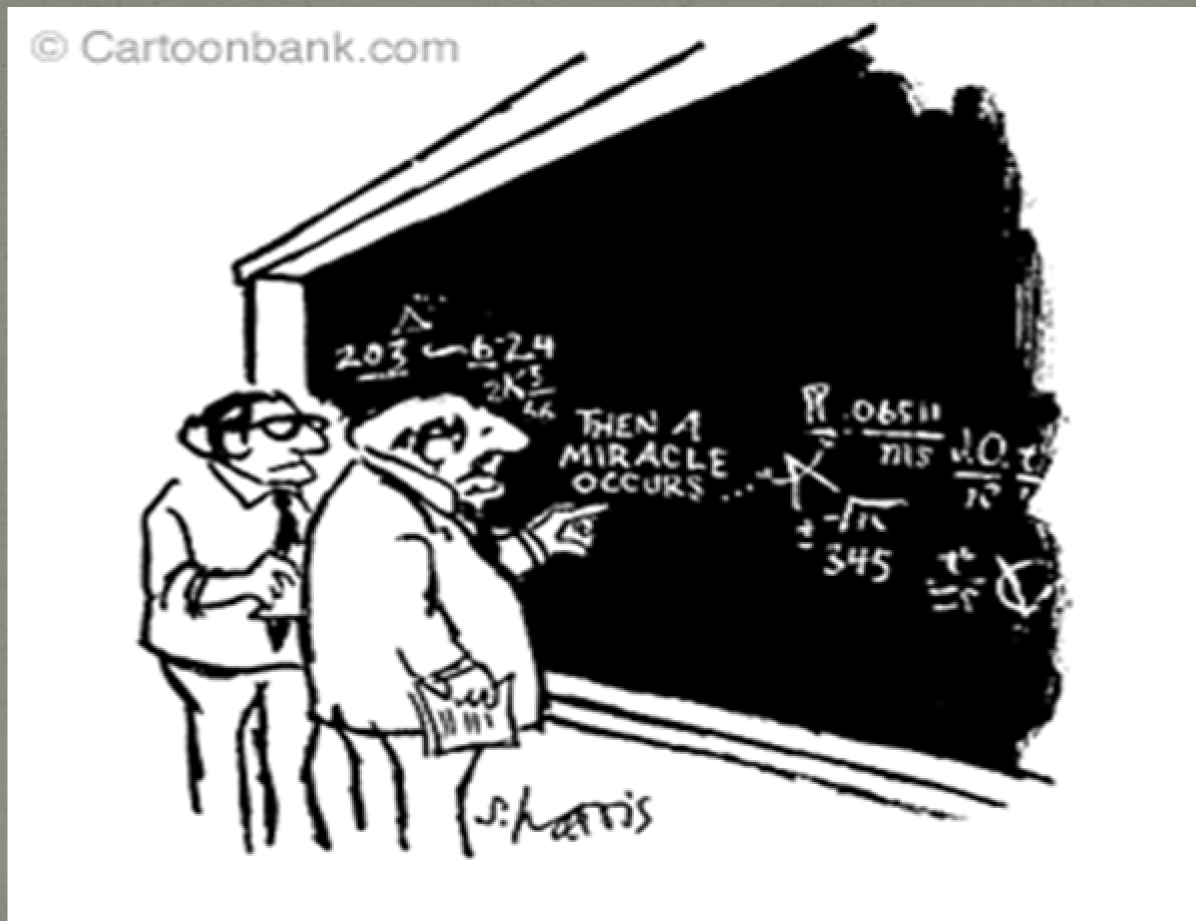
Is it possible that there are hundreds of thousands of people infected with vCJD that don't know it yet?



Conclusions

- Actively engage students
- Suitable curriculum
  - Problems, activities, etc
- Making connections is critical
  - Students understand the “big picture”

QUESTIONS???



"I think you should be more explicit here in step two."