Math 3001 **Due Wed Oct 26** **Assignment #6**

1. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of positive numbers. Prove that

   A)

$$\text{liminf}\frac{a_{n+1}}{a_n} \le \text{liminf}\, a_n^{1/n} \le \text{limsup}\, a_n^{1/n} \le \text{limsup}\frac{a_{n+1}}{a_n}$$

   B)

$$\lim_{n\to\infty} a_n^{1/n} = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$$

   provided they both exist;

2. Give (if possible) an example of a power series which has the following interval of convergence

   a) (-1,1]

   b) [1,-1)

   c) $(-1/2, 0) \cup (0, 1/2)$

   d) [2,4]

3. Let R be the radius of convergence for the power series $\sum a_n x^n$. If infinitely many of the coefficients a_n are nonzero integers, prove that $R \le 1$.

4. Suppose that the series $\sum a_n$ diverges, but the sequence $\{a_n\}_{n=1}^{\infty}$ is bounded. What can you say about the radius of convergence of the power series $\sum a_n x^n$?

5. Prove that the series $\sum a_n x^n$ and $\sum n a_n x^n$ have the same radius of convergence.

6. Let $f_n(x) = x + \frac{1}{n}$ and $f(x) = x$ for $x \in R$.

   a) Show that (f_n) converges uniformly to f.

   b) Show that $(f_n)^2$ converges pointwise to f^2, but not uniformly.

   c) Is it true or false that $(f_n)^2$ converges uniformly to f^2 on any finite segment, i.e. $x \in [a, b]$.

7. Give an example (if possible) of a sequence of functions $f_n(x)$ pointwise convergent to $f(x)$ on (a, b) such that

   a) all $f_n(x)$ are continuous, but $f(x)$ is not.

   b) $\lim_{n\to\infty} f_n'(x) \ne f'(x)$.

   c) $\lim_{n\to\infty} \int_a^b f_n(x)\,dx \ne \int_a^b f(x)\,dx$.

8. EXTRA POINTS

   Suppose (f_n) converges poinwise to f on a set S. Prove that (f_n) converges uniformly to f on every **finite** subset of S.