Math 3001 **Due Fri Sept 23** **Assignment #2**

1. Find the area and the perimeter of the fractal called Koch snowflake.
2. Let f be a bounded function on $[a, b]$. Show that $L(f, P) \leq U(f, Q)$ for any two partitions P and Q of the segment.
3. Let $f(x) = x^3$. Consider partition $P_n = (0,\ 1/n,\ 2/n, ...n/n)$ of $[0, 1]$.
   Find $L(f, P_n)$, $U(f, P_n)$, $L(f)$, $U(f)$, and $\int_0^1 f(x)dx$.
4. Give an example of a function which is not integrable on $[a, b]$, but f^2 is integrable on $[a, b]$.
5. Let f be continuous and non-negative on $[a, b]$. Show that if $L(f) = 0$ then $f(x) = 0$ for all $x \in [a, b]$.
6. Let S be a finite set of points on $[a, b]$. Let f be bounded and $f(x) = 0$ for all x outside from S. Show that f is integrable and $\int_a^b f(x)dx = 0$.
7. Define $F : [0, 1] \to R$ by $F(x) = x$, if x is rational, and $F(x) = 0$ if x is irrational.
   a) Show that $U(f, P) > 1/2$ for any partition P.
   b) Show that $\lim_{n\to\infty} U(f, P_n) = 1/2$ for $P_n = (0,\ 1/n,\ 2/n, ...n/n)$.
   c) Is the function integrable?
8. **Extra Points Problem**
   a) For which functions, if any, $|\int_a^b f(x)dx| = \int_a^b |f(x)|dx$?
   b) For which functions, if any, $|\int_a^b f(x)dx| > \int_a^b |f(x)|dx$?
   c) For which functions, if any, $|\int_a^b f(x)dx| < \int_a^b |f(x)|dx$?