Dear students of M3260 Winter 04,

This is a sample exam from last year. It is a good idea to solve it to check yourself. When you find a solution you can always substitute it into equation to make sure you did it right.

Our exam will be similar to our homework problems. Thus it will also cover systems of linear ODEs with constant coefficients. Instead problems with parametres (like #4 of this test) will be less represented in our exam.

Margo.

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Final **MATHEMATICS 3260** FALL, 2003

Name ID Number:

Notes:

- This exam has 4 questions, appearing on 7 pages.
- The penalty for any and all forms of cheating on this exam will be a grade of zero for the exam.
- You have 2.5 hours to finish.

Marks

[15] 1. Find the general solution for each of the following differential equations.

i) $y' = y + 2\,t\,(y - e^t)$

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ii) $(1 + y)\ \ln(1 + y)\,dt + dy = 0, \quad y(0) = 2$

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iii) $(1 + t^2)\,y'' + 2\,t\,y' = 0$

iv) $y'' + 3\,y' + 2\,y = 3\,t\,e^{-t}$

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v) $y^{(4)} - 2\,y''' + 2\,y'' = 0$

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[6] 2. Use the method of variation of parameters OR reduction of order to find the general solution for

$$y'' - 2\,y' + y = \frac{e^t}{t^2}$$

3.

[2] (a) Define the Laplace transform $\mathcal{L}\{f(t)\}$.

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[4] (b) Derive the expression for $\mathcal{L}\{y''(t)\}$

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[6] (c) Evaluate $\mathcal{L}^{-1}\{\frac{2\,s+1}{s\,(s^2+4)}+\frac{1}{s\,(s-3)}\}$

[7] (d) Use the Laplace transform method to solve

$$y'' + 9\,y = t\,u_2(t)\,, \quad y(0) = 0,\; y'(0) = 1$$

4. Consider a forced but undamped mechanical system described by

$$y'' + y = \sin(\omega t), \quad y(0) = 0,\ y'(0) = 0$$

where ω is a given real number.

[5] (a) Find the solution (i.e. the displacement $y = y(t)$) when $\omega \neq 1$.

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[3] (b) Discuss the solution $y = y(t)$ obtained in (a) when $\omega \neq 1$ but $|\omega - 1|$ is very small. ($\omega \longrightarrow 1$).

[4] (c) What's the solution $y = y(t)$ of the same system when $\omega = 1$?

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[8] (d) Suppose now that the system is damped,

$$y'' + \gamma y' + y = \sin t\,, \quad y(0) = 0\,, y'(0) = 0$$

Determine its motion when the damping coefficient is $\gamma = 2$. Discuss the asymptotic state of the motion (i.e. $\lim_{t\to\infty} y(t)$).

[60]

Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1	$\frac{1}{s}, \quad s > 0$
$e^{a\,t}$	$\frac{1}{s-a}, \quad s > a$
t^n (n is positive integer)	$\frac{n!}{s^{n+1}}, s > 0$
$\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0$
$\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0$
$e^{at}\ \sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$
$e^{at}\ \cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$
$t^n\, e^{at}$ (n is positive integer)	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
$u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
$u_c(t)\, f(t-c)$	$e^{-cs}\, F(s)$
$e^{ct}\, f(t)$	$F(s-c)$