AMAT 3260 assignment #5, due March 19

Problem 1 Using integration by parts, find the Laplace transform of the following functions, where a is a real constant :

1. $t\cos(at)$
2. $t\sinh(at)$, where $\sinh(at) = \dfrac{e^{at} - e^{-at}}{2}$
3. $t^n e^{at}$, $n = 1, 2, 3, ...$

Problem 2 Use the Laplace transform to solve the following initial value problems: Solve the same problem by any other method and compare the results.

1. $y'' - 2y' - 2y = 0; \qquad y(0) = 1, \quad y'(0) = 0$
2. $y'' - 2y' + 2y = \sin(t); \qquad y(0) = 1, \quad y'(0) = 0$
3. $y'' + 2y' + y = e^{-t}; \qquad y(0) = 0, \quad y'(0) = 1$

Problem 3 Find the inverse Laplace transform for the following functions:

1. $\dfrac{2s+2}{s^2+2s+5}$
2. $\dfrac{(s-1)e^{-2s}}{s^2-4s+3}$
3. $\dfrac{e^{-2s}}{s^2-4}$
4. $\dfrac{e^{-s}+e^{s}}{s}$

Problem 4 Find the solutions of the given initial value problems:

1. $y'' + y = \begin{cases} 1, & 0 \leq t < \pi \\ 0, & \pi \leq t < \infty \end{cases}$; $\quad y(0) = 1, \quad y'(0) = 0$

2. $y'' + y = \begin{cases} \sin(t) & 0 \leq t < \pi \\ 0 & t \geq \pi \end{cases}$; $\quad y(0) = 0, \quad y'(0) = 1$

3. $y'' + 4y = \begin{cases} \sin(t) & 0 \leq t < \pi \\ \sin(t) + \sin(t - \pi) & t \geq \pi \end{cases}$; $\quad y(0) = 0, \quad y'(0) = 0$

4. $y'' + 2y' + 2y = \delta(t - 2\pi)$; $\quad y(0) = 1, \quad y'(0) = 0$

5. $y'' + 2y' + 3y = \sin(t) + \delta(t - \pi)$; $\quad y(0) = 0, \quad y'(0) = 0$

6. $y^{(\mathrm{iv})} - y = \delta(t - 1)$; $\quad y(0) = y'(0) = y''(0) = y'''(0) = 0$

Problem 5 Setup and solve the equation of motion for an undamped oscillator with $m = k = 1$ and the external force given by set of impulses $f(t) = \delta(t - \pi) + \delta(t - 2\pi) + \delta(t - 3\pi)$, where at time $t = 0$ the oscillator was at rest.