AMAT 3260 assignment #3, due February 20

Problem 1 Find the solution of the given initial value problems. Describe the behavior as t increases.

1. $y'' + y' - 2y = 0, \quad y(0) = 0,\ y'(0) = 1$
2. $y'' - y = 0, \quad y(0) = 1,\ y'(0) = -1$
3. $-2y'' + y' + y = 0, \quad y(1) = 0,\ y'(1) = 1$
4. $y'' - 2y' - y = 0, \quad y(0) = 0,\ y(0) = -1$

Problem 2 Verify that $y_1 = t^2$ and $y_2 = t^{-1}$ are solutions of the differential equation $t^2y'' - 2y = 0$ for $t > 0$. Show that $\alpha y_1 + \beta y_2$ is also a solution of this equation for any constants α, β.

Problem 3 If the Wronskian W of f and g is $4e^{3t}$, and if $f(t) = e^t$, find $g(t)$.

Problem 4 Find the Wronskian of two solutions of the following equation without actually solving the equation

$$\sin(t)\, y'' + \cos(t)\, y' - t^2 y = 0.$$

Problem 5 Find the solution for each of the following initial value problems. Describe the behavior as t increases.

1. $y'' + 4y = 0, \quad y(0) = 1,\ y'(0) = 0$
2. $y'' + y' + 2y = 0, \quad y(0) = 2,\ y'(0) = -2$
3. $y'' + 2y' + 5y = 0, \quad y(0) = 0,\ y'(0) = 1$
4. $y'' + 2y' + 6y = 0, \quad y(0) = 2,\ y'(0) = 2$

Problem 6 Find the *general solution* and a *fundumental set of solutions* for the given differential equations.

1. $y'' + 2y' + y = 0$
2. $y'' + 6y' + 9y = 0$
3. $-16y'' + 8y' - y = 0$

Problem 7 Solve the following initial value problems.

1. $4y'' - 12y' + 9y = 0, \quad y(0) = 2,\ y'(0) = 1$
2. $4y'' - 4y' + y = 0, \quad y(-1) = 0,\ y'(-1) = 1$
3. $y'' - y' + \frac{1}{4}y = 0, \quad y(1) = 1,\ y'(1) = 0$

Problem 8 Show that the substitution $x = \ln t$ transforms the following Euler equations into an equation with constant coefficients. Solve the equation.

1. $t^2y'' + 4ty' + 2y = 0,\ t > 0;$
2. $t^2y'' - 4ty' - 6y = 0,\ t > 0.$