Math 2051 W2008 Margo Kondratieva

Final Exam: Apr 17 at 9am- 11:30am in room SN 3042.

PLEASE HAVE YOUR PICTURE ID.

Summary.

0. Prerequisite knowledge.

1. Solving of linear systems of equation. Parametric solutions. Rank of the matrix of coefficients. Homogeneous systems.
Text reference: Book 1.1.-1.3
2. Dot product, projection onto a vector, orthogonality.
Text reference: 4.2
3. Eigenvalues and eigenvectors.
Text reference: 3.3
4. Equations of line and plane.
Text reference: 4.1,4.2
5. Determinant and invertibility.
Text reference: 3.2

1. Linear (vector) spaces and subspaces.

1. Definitions of linear (vector) space and its subspace.
Reference: Notes 1, Book 5.1
2. How to use the definitions to prove of disprove that given collection of vectors is a vector space. Reference: Notes 1, Book 5.1
3. Geometry of linear vector sub-spaces in $\mathbf{R}^3$: origin, lines, planes, entire space.
Reference: Notes 1, Book 5.1
4. Null space, Eigen-space, and column-space of a matrix as vector spaces.
Reference: Notes 1, Book 5.1
5. Linear combination of vectors. Span of vectors.
Reference: Notes 2, Book 5.1,
6. Linear independence of vectors: definition and ways to check.
Reference: Notes 2, Book 5.2
7. Basis and dimension of a vector space. Standard basis.
Reference: Notes 2, Book 5.2
8. Orthogonal basis and orthonormal basis. Gram-Schmidt algorithm for orthogonalization with application in Euclidian space.
Reference: Notes 2,3 Book 5.2, 8.1
9. Coordinates of a vector in a basis: definitions and way to find.
Reference: Notes 2
10. Change of basis. Matrix of coordinate transformation. Meaning of the columns of the matrix of coordinate transformation.
Reference: Notes 2
11. Special properties of orthonormal basis: Pythagorean theorem and expansion theorem.
Reference: Notes 2, Book 5.3
12. Basis in the column space and null space of a matrix. Rank as dimension of column or row space of a matrix.
Reference: lectures, Book 5.4
13. Multiplicity of eigenvalues and criterion for diagonalization.
Reference: lectures, Book 5.5

14. Linear transformations of a vector space. Examples in $\mathbf{R}^2$: rotation, projection, reflection.
Reference: Notes 5, Book 4.4, 2.5

15. Invertibility of linear transformation. Kernel and image of a linear transformation as vector subspaces of $\mathbf{R}^n$. Reference: Notes 5, Book 7.2

16*) Matrix of linear transformation in different bases. Reference: Notes 5

2. Quadratic forms.

1. Definition of quadratic form in n variables. Matrix of a quadratic form.
Reference: Notes 3, Book 8.9

2. Inner product and norm as a generalization of dot product and length of a vector. (Definitions and properties). Triangle and Schwarz inequalities.
Reference: Notes 3, Book 5.3, 10.1

3. Squared Norm as a positive definite quadratic form.
Reference: Notes 3,4

4. Symmetric and orthogonal matrices: definitions.
Reference: Notes 4, Book 8.2

5. Special properties of eigenvalues and eigenvectors of symmetric matrices.
Reference: Notes 4, Book 8.2

6. Diagonalization of quadratic form. Principle axis theorem.
Reference: Notes 4, Book 8.2

7. Quadratic forms in two variables. General equation of ellipse and hyperbola.
Reference: Notes 4, Book 8.9

8. Canonical equation of ellipse and hyperbola and their graphs.
Reference: Notes 4

9. Sketching ellipse and hyperbola given by general equations. Axes of symmetry, asymptotes, vertices.
Reference: Notes 4, Book 8.9

10. Change of variables (rotation) and geometry of the process of diagonalization of a quadratic form in 2D.
Reference: Notes 4, Book 8.9

11. Unit ball in an inner product space. Euclidian space as a special case. Graphs of a unit ball in $\mathbf{R}^2$ and $\mathbf{R}^3$.
Reference: Notes 4.

References

Book: W.K.Nicholson, Linear Algebra, Fifth edition.
Notes posted on www.math.mun.ca/˜mkondra
Notes 1: Week 1. Linear vector space and subspace.
Notes 2: Week 2-3. Change of basis in a vector space.
Notes 3: Week 8-9. Inner product spaces.
Notes 4: Week 10-11. Quadratic forms. Principal axes theorem.
Notes 5: Last week. Linear transformations.

Good luck!