MATH 2051 MIDTERM EXAM on February 22,2008

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Student:

1. (9 points)

   (a) Complete the definition:
   A linear (vector) space is a collection of vectors with the following properties:
   (1)

   (2)

   (3)

   (b) Use you definition in part (a) to prove that:
   The collection of all vectors $\vec{v} = (x, y, z)^T$ *which are orthogonal to vector*

   $$\vec{n} = (22, 2, 2008)^T$$

   form a linear (vector) space.

2. (9 point)

   (a) Give a definition of n **linearly independent** vectors.

(b) Let the columns of a rectangular $m \times n$ matrix A be linearly independent vectors, and $m \neq n$. Which of the following are TRUE?

(1) $m > n$

(2) $m < n$

(3) linear independence of columns is possible only for $m = n$.

(4) system $AX = B$ has a **unique** solution for **any** $m \times 1$ matrix B.

(5) system $AX = B$ has a **parametric** solution for **some** $m \times 1$ matrices B.

(6) system $AX = B$ has a **unique** solution for **some** $m \times 1$ matrices B.

(c) Give an example of a matrix with four linearly independent rows. What is the rank of your matrix?

3. (8 point)

Let $\vec{f_1} = \vec{e}_1 + 2\vec{e}_2$, $\vec{f_2} = \vec{3}e_1 + 4\vec{e}_2$, and $\vec{v} = 5\vec{e}_1 + 6\vec{e}_2$,

(a) Find the matrix of coordinate transformation for the change from basis $(\vec{e}_1, \vec{e}_2)$ to basis $(\vec{f_1}, \vec{f_2})$;

(b) Find the matrix of coordinate transformation for the change from basis $(\vec{f_1}, \vec{f_2})$ to basis $(\vec{e}_1, \vec{e}_2)$;

(c) Find numbers a, b such that $\vec{v} = a\vec{f_1} + b\vec{f_2}$.

4. (9 points)

(a) Give a definition of **basis** in a vector space.

(b) Find a basis in the **column space** of matrix $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 6 & 9 & 12 & 13 & 14 \end{bmatrix}$

(c) Find a basis in the **eigenspace** of matrix $\begin{bmatrix} 2008 & 2008 & 2008 \\ 0 & 2008 & 2008 \\ 0 & 0 & 2008 \end{bmatrix}$

5. (14 points)

(a) Let $\vec{v}_1, \vec{v}_2, \vec{v}_3$ be an orthonormal basis in a vector space S. Let (x, y, z) denote coordinates of a vector $\vec{w} \in S$ w.r.t this basis. Prove that

$$x = \vec{w} \cdot \vec{v}_1, \quad y = \vec{w} \cdot \vec{v}_2, \quad z = \vec{w} \cdot \vec{v}_3.$$

(b) Show that the following set of vectors is an **orthogonal basis** in space $S = span(\vec{f_1}, \vec{f_2}, \vec{f_3})$

$$\vec{f_1} = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \quad \vec{f_2} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ -4 \end{bmatrix}, \quad \vec{f_3} = \begin{bmatrix} 0 \\ 3 \\ 0 \\ 0 \end{bmatrix}.$$

(c) Obtain **orthonormal basis** from vectors $\vec{f_1}, \vec{f_2}, \vec{f_3}$ given in (b), by normalization. Call the vectors you found $\vec{v_1}, \vec{v_2}, \vec{v_3}$.

(d) Find coordinates of $\vec{w} = (4, 9, 4, -7)^T$ w.r.t the orthonormal basis $\vec{v_1}, \vec{v_2}, \vec{v_3}$ found in (c).