Math 2051 W2008 Margo Kondratieva

Exercise Set 2 for Quiz on Fri Jan 25.

1. Let n be an integer greater than 1. Give a definition of:
   -linear combination of n vectors;
   -span of n vectors;
   -n linearly dependent vectors;
   -n linearly independent vectors;
   -basis of a vector space;
   -dimension of a vector space.

2. Give an example of:
   -linear combination of two vectors;
   -span of three vectors;
   -four linearly dependent vectors;
   - five linearly independent vectors;
   -basis of a vector space;
   -a vector space of dimension one and its basis.

3. Are the following triples of vectors linearly dependent or linearly independent? Justify.

   a) $\vec{u} = (1,2)^T$, $\vec{v} = (2,2)^T$, $\vec{w} = (3,1)^T$.

   b) $\vec{u} = (1,2,3)^T$, $\vec{v} = (4,5,6)^T$, $\vec{w} = (7,8,9)^T$.

   c) $\vec{u} = (1,2,0)^T$, $\vec{v} = (1,0,2)^T$, $\vec{w} = (0,1,2)^T$.

   d) $\vec{u} = (1,0,0)^T$, $\vec{v} = (2,3,0)^T$, $\vec{w} = (3,4,5)^T$.

4. Find dimension and basis of the linear vector space defined as

   a) span $\{\vec{u} = (1,2,3)^T, \vec{v} = (3,2,1)^T, \vec{w} = (8,8,8)^T\}$

   b) null A, where matrix $A = \begin{bmatrix} 2 & 3 & 4 & 5 & 6 \\ 0 & 0 & 0 & 3 & 4 \end{bmatrix}$.

   c) eigenspace of A, where $A = \begin{bmatrix} 10 & 20 & 30 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$.