Math 2051 W2008 Margo Kondratieva

Exercise Set 4 for Quiz on Wed Feb 13.

1. Give a definition of:
   -dot product of two vectors given via their coordinates in standard basis; (math 2050)
   -projection of one vector onto another vector (math 2050).
   -orthogonal basis and orthonormal basis (Set 3);
   -column space of a matrix;
   -row space of a matrix;
   -rank of a matrix (math 2050);

2. Explain with examples the meaning of the following objects for solving systems of linear equations:
   -column space of a matrix;
   -row space of a matrix;
   -rank of a matrix;

3. Find basis in column space, row space and null space of matrix A. Give an example of vector B for which the system $AX = B$ has a solution, and find all the solutions corresponding to B.

$$A = \begin{bmatrix} 1 & -1 & 5 & -2 & 2 \\ 2 & -2 & -2 & 5 & 1 \\ 0 & 0 & -12 & 9 & -3 \\ -1 & 1 & 7 & -7 & 1 \end{bmatrix}.$$

4. Find basis in span of vectors

$$v_1 = \begin{bmatrix} 1 & 5 & -6 \end{bmatrix}^T, \quad v_2 = \begin{bmatrix} 2 & 6 & -8 \end{bmatrix}^T, \quad v_2 = \begin{bmatrix} 3 & 7 & -10 \end{bmatrix}^T, \quad v_4 = \begin{bmatrix} 4 & 8 & 12 \end{bmatrix}^T.$$

5. Explain whether a 5×7 matrix can have:
   -linearly independent columns;
   -linearly independent row;
   -rank 6;
   -null space of dimension 6;

6. Explain why the following is true:
   a) $|\vec{u} \cdot \vec{v}| \leq |\vec{u}||\vec{v}|$ (Cauchy inequality).
   b) $|\vec{u} + \vec{v}| \leq |\vec{v}| + |\vec{v}|$ (Triangle inequality).
   c) If $\vec{v}_1, ..., \vec{v}_n$ is an orthonormal basis then any $\vec{w} \in span(\vec{v}_1, ..., \vec{v}_n)$ has coordinates $a_j = \vec{w} \cdot \vec{v}_j$, $j = 1, 2, ...n$, in this basis.
   d) dim(col$A$)=rank(A);
   e) If A is $m \times n$ and rank $A = n$ then $A^T A$ is invertable.
   f) If A is $m \times n$ and rank $A = n$ then $CA = I$, for some $n \times m$ matrix C. Here I is $n \times n$ identity matrix.