1. (a) "No". For example, if $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, then $AB = 0$.

   (b) Since $AB = \begin{bmatrix} a & 2a-b \\ c & 2c-d \end{bmatrix}$, if $AB = 0$, then $a = 2a - b = c = 2c - d = 0$, so $a = b = c = d = 0$ and $A = 0$. This result does not contradict part (a) which says that $AB = 0$ need not **always** force $A = 0$ or $B = 0$.

   (c) If $XB = YB$, then $(X - Y)B = 0$ so, by part (b), we conclude that $X - Y = 0$. So $X = Y$.

2. Substituting $x = \frac{\pi}{3}$, $y = 2$, gives $a\frac{\sqrt{3}}{2} + \frac{1}{2}b = 2$. Substituting $x = -\frac{\pi}{4}$, $y = 1$ gives $-\frac{\sqrt{2}}{2}a + \frac{\sqrt{2}}{2}b = 1$. Thus
$$\begin{aligned} \sqrt{3}a + \phantom{\sqrt{2}}b &= 4 \\ -\sqrt{2}a + \sqrt{2}b &= 2. \end{aligned}$$

   In matrix form, this is $\begin{bmatrix} \sqrt{3} & 1 \\ -\sqrt{2} & \sqrt{2} \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$.

3. $A + B = \begin{bmatrix} 1 & 1 \\ 8 & 2 \end{bmatrix}$, so $(A+B)^2 = \begin{bmatrix} 9 & 3 \\ 24 & 12 \end{bmatrix}$.

   $A^2 = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$, $B^2 = \begin{bmatrix} -5 & 2 \\ -10 & -1 \end{bmatrix}$, $AB = \begin{bmatrix} 10 & -5 \\ 20 & -11 \end{bmatrix}$ and so
$$A^2 + 2AB + B^2 = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} + \begin{bmatrix} 20 & -10 \\ 40 & -22 \end{bmatrix} + \begin{bmatrix} -5 & 2 \\ -10 & -1 \end{bmatrix} = \begin{bmatrix} 22 & 2 \\ 45 & -1 \end{bmatrix}.$$

   No, $(A+B)^2 \neq A^2 + 2AB + B^2$. The correct expansion is $(A+B)^2 = (A+B)(A+B) = A^2 + AB + BA + B^2$.

4. Neither A nor B has an inverse; neither is square. This is an example where $AB = I$ but $BA \neq I$.

5. If $X^{-1}AY^{-1} = B$, then $A = XBY$ (multiplying by X on the left and by Y on the right). So
$$A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} -3 & 4 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 8 \\ -9 & 12 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -23 & 5 \\ -57 & 3 \end{bmatrix}.$$

6. Since x is $n \times 1$, x^T is $1 \times n$, so $\mathsf{x}^T A\mathsf{x}$ is the product of a $1 \times n$, an $n \times n$ and an $n \times 1$ matrix. This is 1×1.

7. $(AB)^T = B^TA^T$, so $((AB)^T)^{-1} = (B^TA^T)^{-1} = (A^T)^{-1}(B^T)^{-1}$.