1. The answer is an equation of the form $18x + 6y - 5z = d$. Substituting $x = -1$, $y = 1$, $z = 7$, we get $d = -18 + 6 - 35 = -47$, so the plane has equation $18x + 6y - 5z = -47$.

2. $\overrightarrow{AB} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ and $\overrightarrow{AC} = \begin{bmatrix} 8 \\ -5 \\ -1 \end{bmatrix}$. A normal vector is $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathsf{i} & \mathsf{j} & \mathsf{k} \\ 1 & -1 & 0 \\ 8 & -5 & -1 \end{vmatrix} = 1\mathsf{i} - (-1)\mathsf{j} + 3\mathsf{k} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$. The plane has equation of the form $x + y + 3z = d$. Since the coordinates of A satisfy the equation, we have $-1 + 2 + 3 = d$, so $d = 4$ and the equation is $x + y + 3z = 4$.

3. First note that the lines are parallel since the direction of one is a scalar multiple of the direction of the other: $\begin{bmatrix} 2 \\ -10 \\ -4 \end{bmatrix} = -2\begin{bmatrix} -1 \\ 5 \\ 2 \end{bmatrix}$. We can conclude that the lines are the same if, in addition, they have a point in common. So we look for a solution to

$$\begin{aligned} -1 - t &= 1 + 2s \\ 4 + 5t &= -6 - 10s \\ 4 + 2t &= -4s; \end{aligned} \qquad \text{that is, to} \qquad \begin{aligned} 2s + t &= -2 \\ 10s + 5t &= -10 \\ 4s + 2t &= -4 \end{aligned}$$

This system is equivalent to $2s + t = -2$ which has infinitely many solutions; e.g., $t = 0, s = -1$. This gives the point $(-1, 4, 4)$.

4. (a) The line has direction $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and the plane has normal $\begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix}$. Since the dot product of these vectors is $-2 \neq 0$, they are not perpendicular. Hence the line and plane are not parallel, so they must intersect.

   (b) A point (x, y, z) is on the line if $x = 2 + t$, $y = -3 + 2t$, $z = -4 + 3t$ for some t. Substituting into the equation of the plane gives

$$3(2 + t) - 4(-3 + 2t) + (-4 + 3t) = 18 = -2t + 14.$$

   Thus $2t = -4$, $t = -2$ and the point of intersection is $(0, -7, -10)$.

5. The projection of u on v is $\operatorname{proj}_{\mathsf{v}} \mathsf{u} = \dfrac{\mathsf{u} \cdot \mathsf{v}}{\mathsf{v} \cdot \mathsf{v}} \mathsf{v} = \frac{32}{77}\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$.

   The projection of v on u is $\operatorname{proj}_{\mathsf{u}} \mathsf{v} = \dfrac{\mathsf{v} \cdot \mathsf{u}}{\mathsf{u} \cdot \mathsf{u}} \mathsf{u} = \frac{32}{14}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

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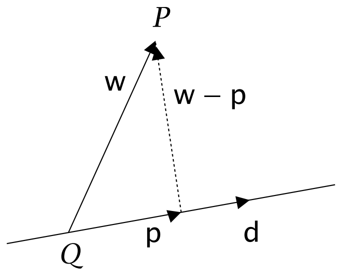
6. Let Q be any point on the line, say $Q(1, 2, 3)$. The line has direction $\mathsf{d} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and the distance we want is the length of $\mathsf{w} - \mathsf{p}$, where $\mathsf{p} = \operatorname{proj}_{\mathsf{d}} \mathsf{w}$ is the projection of $\mathsf{w} = \overrightarrow{QP} = \begin{bmatrix} -2 \\ 0 \\ -2 \end{bmatrix}$ on d.



We have

$$\mathsf{p} = \operatorname{proj}_{\mathsf{d}} \mathsf{w} = \frac{\mathsf{w} \cdot \mathsf{d}}{\mathsf{d} \cdot \mathsf{d}} \mathsf{d} = -\frac{4}{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

and

$$\mathsf{w} - \mathsf{p} = \begin{bmatrix} -2 \\ 0 \\ -2 \end{bmatrix} + \frac{4}{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$$

so the required distance is $\frac{2}{3} \left\| \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \right\| = \frac{2}{3}\sqrt{6}$.