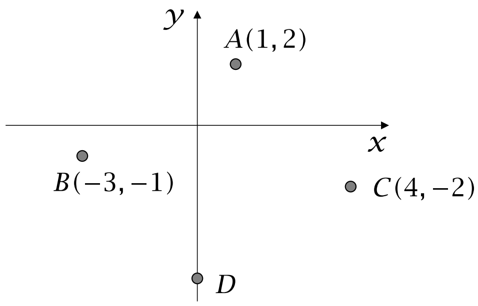
1. We are given $\mathsf{u}\cdot\mathsf{u} = 3^2 = 9$ and $\mathsf{v}\cdot\mathsf{v} = 5^2 = 25$. Thus $(-3\mathsf{u}+4\mathsf{v})\cdot(2\mathsf{u}+5\mathsf{v}) = -6\mathsf{u}\cdot\mathsf{u} - 7\mathsf{u}\cdot\mathsf{v} + 20\mathsf{v}\cdot\mathsf{v} = -6(9) - 7(8) + 20(25) = 390$.

2. The length of v is $\|\mathsf{v}\| = \sqrt{(-1)^2+2^2+2^2} = 3$, so $\frac{1}{3}\begin{bmatrix}-1\\2\\2\end{bmatrix} = \begin{bmatrix}-\frac{1}{3}\\\frac{2}{3}\\\frac{2}{3}\end{bmatrix}$ is a unit vector in the direction of v and $\frac{2}{3}\begin{bmatrix}-1\\2\\2\end{bmatrix} = \begin{bmatrix}-\frac{2}{3}\\\frac{4}{3}\\\frac{4}{3}\end{bmatrix}$ is a vector of length 2 in the direction of v. The vector $-6(\frac{1}{3})\begin{bmatrix}-1\\2\\2\end{bmatrix} = \begin{bmatrix}2\\-4\\-4\end{bmatrix}$ has length 6 and opposite direction.

3. $\mathsf{u}\cdot\mathsf{v} = -4-3+2 = -5$, $\|\mathsf{u}\| = \sqrt{14}$ and $\|\mathsf{v}\| = \sqrt{18}$, so $\cos\theta = \dfrac{\mathsf{u}\cdot\mathsf{v}}{\|\mathsf{u}\|\,\|\mathsf{v}\|} = -\dfrac{5}{\sqrt{14}\sqrt{18}} \approx -.315$. Thus $\theta = \arccos(-.315) \approx 1.89$ rads $\approx 108°$.

4. We wish to find k so that the dot product of u and $\mathsf{u}+k\mathsf{v} = \begin{bmatrix}1+2k\\k\\-1\end{bmatrix}$ is 0. Since $\mathsf{u}\cdot(\mathsf{u}+k\mathsf{v}) = \begin{bmatrix}1\\0\\-1\end{bmatrix}\cdot\begin{bmatrix}1+2k\\k\\-1\end{bmatrix} = 1+2k+1$, we find $k=-1$.

5. The picture at the right shows the approximate position of the points. There is only one possible location for D. Let D have coordinates (x,y). Since $\overrightarrow{CA} = \begin{bmatrix}-3\\4\end{bmatrix} = \overrightarrow{DB}$, we need $\begin{bmatrix}-3\\4\end{bmatrix} = \begin{bmatrix}-3-x\\-1-y\end{bmatrix}$, $-3-x=-3$ and $4=-1-y$. Thus $x=0$, $y=-5$ and D is $(0,-5)$.



Since $\overrightarrow{CA}\cdot\overrightarrow{DC} = \begin{bmatrix}-3\\4\end{bmatrix}\cdot\begin{bmatrix}4\\3\end{bmatrix} = 0$, there is a right angle at C and since $\overrightarrow{CA}\cdot\overrightarrow{BA} = \begin{bmatrix}-3\\4\end{bmatrix}\cdot\begin{bmatrix}4\\3\end{bmatrix} = 0$, there is a right angle at A. Thus $ABDC$ is a rectangle and since $\|\overrightarrow{BA}\| = \|\overrightarrow{AC}\| = 5$, the rectangle is a square.

6. Since $\mathsf{u} \times \mathsf{v} = \begin{bmatrix} u_2v_3 - u_3v_2 \\ -(u_1v_3 - v_1u_3) \\ u_1v_2 - u_2v_1 \end{bmatrix}$, we have $(\mathsf{u} \times \mathsf{v}) \cdot \mathsf{u}$

$$\begin{aligned} &= (u_2v_3 - u_3v_2)u_1 - (u_1v_3 - v_1u_3)u_2 + (u_1v_2 - u_2v_1)u_3 \\ &= u_2v_3u_1 - u_3v_2u_1 - u_1v_3u_2 + v_1u_3u_2 + u_1v_2u_3 - u_2v_1u_3 = 0. \end{aligned}$$

7. $a\mathsf{u} + b\mathsf{v} = \begin{bmatrix} 3a + 4b \\ 2a - b \\ a - b \end{bmatrix}$. A normal is

$$\begin{aligned} \mathsf{u} \times \mathsf{v} &= \begin{vmatrix} \mathsf{i} & \mathsf{j} & \mathsf{k} \\ 3 & 2 & 1 \\ 4 & -1 & -1 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ -1 & -1 \end{vmatrix} \mathsf{i} - \begin{vmatrix} 3 & 1 \\ 4 & -1 \end{vmatrix} + \begin{vmatrix} 3 & 2 \\ 4 & -1 \end{vmatrix} \\ &= -1\mathsf{i} - (-7)\mathsf{j} + (-11)\mathsf{k} = \begin{bmatrix} -1 \\ 7 \\ -11 \end{bmatrix}. \end{aligned}$$

The vector $\begin{bmatrix} -1 \\ 7 \\ -11 \end{bmatrix}$ is a normal. Since the plane in question passes through $(0, 0, 0)$, an equation is $-x + 7y - 11z = 0$.