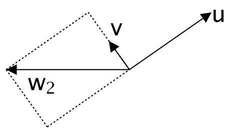
Assignment 1 | **MATH 2050 SOLUTIONS** | Fall 2004

1. Let $A = \begin{bmatrix} x \\ y \end{bmatrix}$. Then $\overrightarrow{AB} = \begin{bmatrix} 1-x \\ 4-y \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, so $1 - x = -1$, $4 - y = 2$ and $A = (2, 2)$.

2. $\mathsf{x} = \frac{3}{2}\mathsf{u} = -2\mathsf{w} = \frac{2}{5}\mathsf{y}$; $\mathsf{x}$ is not a scalar multiple of $\mathsf{v}$.

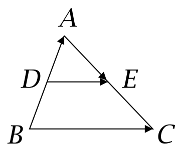
3. (a) $\begin{bmatrix} 2a \\ 5a-6 \end{bmatrix}$

   (b) $2\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + 5\begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix} + 3\begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -8 \\ 7 \\ 6 \end{bmatrix}$.

4. 

It appears that $\mathsf{w}_2 = -\mathsf{u} + 2\mathsf{v}$.

5. We wish to show that $\overrightarrow{DE} = \frac{1}{2}\overrightarrow{BC}$. Now $\overrightarrow{AD} = \frac{1}{2}\overrightarrow{AB}$ and $\overrightarrow{AE} = \frac{1}{2}\overrightarrow{AC}$, so $\overrightarrow{DE} = \overrightarrow{DA} + \overrightarrow{AE} = -\frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AC} = \frac{1}{2}(\overrightarrow{AC} - \overrightarrow{AB}) = \frac{1}{2}\overrightarrow{BC}$.




6. We wish to find $a$ and $b$ such that $\begin{bmatrix} 7 \\ 7 \end{bmatrix} = a\begin{bmatrix} -1 \\ 1 \end{bmatrix} + b\begin{bmatrix} 5 \\ 2 \end{bmatrix}$.

   Thus, we should have $\begin{aligned} -a + 5b &= 7 \\ a + 2b &= 7 \end{aligned}$.

   Adding these equations gives $7b = 14$, so $b = 2$ and $a = 7 - 2b = 3$.

   We obtain $\begin{bmatrix} 7 \\ 7 \end{bmatrix} = 3\begin{bmatrix} -1 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 5 \\ 2 \end{bmatrix}$

7. A linear combination of $\mathsf{u}$ and $\mathsf{v}$ is a vector of the form $a\mathsf{u} + b\mathsf{v}$. This is the same as $\frac{1}{2}a(2\mathsf{u}) + (-\frac{1}{3}b)(-3\mathsf{v})$, which is a linear combination of $2\mathsf{u}$ and $-3\mathsf{v}$.

8. We are given that $a\mathsf{u} + b\mathsf{v} = \mathsf{0}$ for certain numbers $a$, $b$ which are not both 0. Suppose $a \neq 0$. Then $a\mathsf{u} = -b\mathsf{v}$ implies $\mathsf{u} = -\frac{b}{a}\mathsf{v}$, which says that $\mathsf{u}$ is a multiple of (and so parallel to) $\mathsf{v}$. A similar argument applies if $b \neq 0$.

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