MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 8 **MATH 2050 sect. 3** DUE FRI NOV. 10.

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1. Find the characteristic polynomial, eigenvalues, eigenvectors and (if possible) an invertable matrix P such that $P^{-1}AP$ is diagonal.

   Hint: all eigenvalues in this problem are integers.

   (a) $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 7 & 0 & -4 \\ 0 & 5 & 0 \\ 5 & 0 & -2 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 6 & -6 \\ 1 & 2 & -1 \end{bmatrix}$

   (d) $A = \begin{bmatrix} 3 & 1 & 1 \\ -4 & -2 & -5 \\ 2 & 2 & 5 \end{bmatrix}$

2. Given $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, show that

   a) the characteristic polynomial is $x^2 - trA \cdot x + \det A$ (recall $trA = a + d$).

   b) the eigenvalues are
$$\frac{a+d}{2} \pm \sqrt{\left(\frac{a-d}{2}\right)^2 + bc}$$

3. If $P^{-1}AP$ and $P^{-1}BP$ are both diagonal, show that $AB = BA$.

4. Suppose λ is an eigenvalue of a square matrix A. Show that λ^2 is an eigenvalue of A^2 (with the same eigenvector X).

   What can you conjecture (and prove) about λ^k and A^k for any $k \geq 2$?

5. Consider a linear dinamical sytem $V_{k+1} = AV_k$ for $k \geq 0$. Find exact formula for V_k. Approximate V_k for large values of k.

   (a) $A = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$, $V_0 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 3 \\ 1 & 4 & 1 \end{bmatrix}$ $V_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$