MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 8 | **MATH 2050** | ANSWERS

For practice and extra credit only.

1. Consider an arbitrary quadrilateral in a plane. Connect the midpoints of its sides to get a new quadrilateral. What do you notice about this new quadrilateral? Prove you conjecture using addition of vectors.

   Hint: start from looking at midpoints of various squares and rectangles and then take a more general quadrilateral.

   Answer: This queadrilateral is a parallelogram.

2. Find whether two lines with direction vectors $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 3 \\ 3 \end{bmatrix}$ respectively intersect at a point if it is known that the first line goes via point (0,-2,4) and the second via point (1,1,2).

   Answer: No. Because the system of linear equations $t = -s + 1$, $2t - 2 = 3s + 1$, $3t + 4 = 3s + 2$ is inconsistent and thus does not have a solution.

3. Write equation of line going via points (1,2) and (4,5). Does point A belong to the line? If not, what is the distance from the point to the line?

   a) A(11,12)

   b) A(5,7,9)

   Answer: The line has equation $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + t \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$.

   a) point A(11,12,0) is on the line: (t=10/3).

   b) point A(5,7,9) is not on the line. The distance is approximatly 9.03 units.

   Pick point B(1,2,0) on the line then $||\vec{B}A|| = \sqrt{122}$. Cosine of the angle between $\vec{B}A$ and the direction vector $\vec{d} = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$ of the line is $9/\sqrt{244}$. The distance is found from this right triangle.

4. Find the projection of vector $\vec{v} = \begin{bmatrix} 2 \\ -5 \\ 7 \end{bmatrix}$ onto vector $\vec{u} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$.

   Find the angle between the vectors.

   Answer: the projection is $\frac{10}{7} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$; cosine of the angle is $10/\sqrt{273}$; the angle is approximatly 53 degrees.

5. Consider triangle with vertices at points

$$A(1,-1,1), \quad B(3,-6,8), \quad C(1,-2,4),$$

(a) find the area of the triangle

(b) find equation of the plane to which the triangle belongs.

Answer: Calculate the cross product of $\vec{AB} = \begin{bmatrix} 2 \\ -5 \\ 7 \end{bmatrix}$ and $\vec{AC} = \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix}$. The cross product is $\begin{bmatrix} -8 \\ -6 \\ -2 \end{bmatrix}$.

the area of the triangle is the half of the magnitude of the cross product: $\sqrt{26}$. The equation of the plane is $8(x-1)+6(y+1)+2(z-1)=0$ or equivalently, (after devision by 2) $4x+3y+z=2$.

6. Give an example of a plane which is orthogonal to the plane $7x-y-4z=6$.

Answer: $4y-z=1$. The normal vector of the two planes are orthogonal thus the planes themselves are orthogonal.

7. Give an example of a line which is orthogonal to the vector $\begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}$. Explain.

Answer: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} + t \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$.

The direction vector of the line is orthogonal to the given vector.