MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 7 **MATH 2050** DUE THUR JULY 12

1. (a) Give a definition of an eigenvalue and corresponding eigenvector of a matrix.
   (b) Explain the method (step by step) how to find all the eigenvalues and corresponding eigenvectors of a matrix.
   (c) Give an exapmle of a problem when knowing the eigenvalues and eigenvectors of a matrix can be useful to find the solution explicitly.

2. Find the characteristic polynomial, eigenvalues, eigenvectors and (if possible) an invertable matrix P such that $P^{-1}AP$ is diagonal. If the later is not possible, explain why.

   Hint: all eigenvalues in this problem are integers.

   (a) $A = \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 2 & 0 \\ 3 & 2 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 2 & -16 & -2 \\ 0 & 5 & 0 \\ 2 & -8 & -3 \end{bmatrix}$

   (d) $A = \begin{bmatrix} 2 & 1 & -12 \\ 0 & 1 & 11 \\ 1 & 1 & 4 \end{bmatrix}$

3. Let a matrix A have eigenvalues $\lambda_1 = -2$ and $\lambda_2 = 3$ with corresponding eigenvectors $X_1 = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$ and $X_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$. Find the matrix A.

4. Let $A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & -3 & -2 \\ 5 & 5 & 4 \end{bmatrix}$. Find an invertable matrix P such that $P^{-1}AP = \begin{bmatrix} 4 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.

5. Consider a linear dinamical sytem $V_{k+1} = AV_k$ for $k \geq 0$. Find exact formula for V_k. Approximate V_k for large values of k.

   (a) $A = \begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix}$, $V_0 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 2 & 0 & 2 \\ -16 & 5 & -8 \\ -2 & 0 & -3 \end{bmatrix}$ $V_0 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

6. True or False? Explain.

   a) every square matrix is *diagonalizable* (i.e. *similar* to a diagonal matrix).

   b) any $n \times n$-matrix has at most n distinct eigenvalues.

   c) if $\lambda \neq 0$ is an eigenvalue of A and A is invertable then λ^{-1} is an eigenvalue of A^{-1}.