MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 7 **MATH 2050 Sec. 3** DUE WEDNESDAY NOV 7

1. (a) Give a definition of an eigenvalue and corresponding eigenvector of a matrix.
   (b) Explain the method (step by step) how to find all the eigenvalues and corresponding eigenvectors of a matrix.
   (c) Give an exapmle of a problem when knowing the eigenvalues and eigenvectors of a matrix can be useful to find the solution explicitly.

2. Find the characteristic polynomial, eigenvalues, eigenvectors and (if possible) an invertable matrix $P$ such that $P^{-1}AP$ is diagonal. If the later is not possible, explain why.

   *Hint: all eigenvalues in this problem are integers.*

   (a) $A = \begin{bmatrix} 2 & 4 \\ 5 & 3 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 7 & 0 & -4 \\ 0 & 5 & 0 \\ 5 & 0 & -2 \end{bmatrix}$

   (d) $A = \begin{bmatrix} 3 & 1 & 1 \\ -4 & -2 & -5 \\ 2 & 2 & 5 \end{bmatrix}$

3. Let a matrix $A$ have eigenvalues $\lambda_1 = -2$ and $\lambda_2 = 3$ with corresponding eigenvectors $X_1 = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$ and $X_2 = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$. Find the matrix $A$.

4. Let $A = \begin{bmatrix} 1 & 3 & 2 \\ -1 & 2 & 1 \\ 4 & -1 & -1 \end{bmatrix}$. Find an invertable matrix $P$ such that $P^{-1}AP = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$.

5. Consider a linear dynamical system $V_{k+1} = AV_k$ for $k \geq 0$. Find exact formula for $V_k$. Approximate $V_k$ for large values of $k$.

   (a) $A = \begin{bmatrix} 2 & 4 \\ 5 & 3 \end{bmatrix}$, $V_0 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

   (b) $A = \begin{bmatrix} -1 & 3 & -3 \\ -3 & 5 & -3 \\ -6 & 6 & -4 \end{bmatrix}$ $V_0 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$

6. True or False? Explain.

   a) If $\lambda$ is an eigenvalue of matrix $A$ , then $\lambda^{100}$ is an eigenvalue of the matrix $A^{100}$

   b) If we substitute matrix $A$ into its characteristic equation then the result is the identity matrix.

   c) If $\lambda \neq 0$ is an eigenvalue of $A$ then $\lambda^{-1}$ is an eigenvalue of $A^{-1}$.