MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 7 **MATH 2050 sect. 3** DUE: FRIDAY NOVEMBER 3

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1. Evaluate determinant of each matrix by **two** ways:

   (1) using Laplace expansion, and

   (2) by reducing the matrix to the upper triangular form.

   (a) $\begin{bmatrix} 3 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & -1 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & a & b \\ a & b & 1 \\ b & 1 & a \end{bmatrix}$

   (b) $\begin{bmatrix} 2 & 1 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 1 & 3 & -1 & 2 \\ -1 & -1 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \end{bmatrix}$

2. The *characteristic polynomial* of matrix A is defined as $P_A(x) = \det(x \cdot I - A)$, where I is the identity matrix of the same size as A. Find the characteristic polynomial for $A = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$. Evaluate the polynomial for $x = A$.

   (Example: if $p(x) = 3x^2 - 4x + 5$ then for $x = A$ we obtain matrix $p(A) = 3A^2 - 4A + 5I$.

3. Let A, B, C be square matrices of the same size. Given $\det A = -1$, $\det B = 2$, and $\det C = 3$, find $\det A^5 B C^T A^{-1} B^2$.

4. Explain what can be said about $\det A$ if:

   (a) $A^2 = A$

   (b) $A^k = 0$ for some integer k. Here 0 is a zero matrix.

   (c) $A^2 + I = 0$

   (d) $A^3 = A$

   (e) $A = A^T$

   (f) $A^{-1} = A^T$

5. Let a square matrix A was obtained from a square matrix B by a series of elementary row operations. Is it true that $\det A = \det B$? Give an example supporting your answer.

6. Find all values a such that the matrix is invertable

   (a) $\begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & a \\ 2 & a & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & a & -a \\ -1 & 1 & -1 \\ a & -a & a \end{bmatrix}$