MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 6 **MATH 2050** ANSWERS.

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1. For each of the following matrices find:
   1. the matrix of cofactors;
   2. the determinant using the cofactor (Laplace) expansion;
   3. the inverse matrix using the cofactors;

$$A = \begin{bmatrix} 10 & 2 \\ 5 & 4 \end{bmatrix}.$$

1.The matrix of cofactors is $\begin{bmatrix} 4 & -5 \\ -2 & 10 \end{bmatrix}$.

2. $\det A = 30$.

3. $A^{-1} = \frac{1}{30}\begin{bmatrix} 4 & -5 \\ -2 & 10 \end{bmatrix}^T = \frac{1}{30}\begin{bmatrix} 4 & -2 \\ -5 & 10 \end{bmatrix}$.

$$B = \begin{bmatrix} 1 & -1 & 2 \\ -5 & 7 & -11 \\ -2 & 3 & -5 \end{bmatrix}.$$

1.The matrix of cofactors is $\begin{bmatrix} -2 & -3 & -1 \\ 1 & -1 & -1 \\ -3 & 1 & 2 \end{bmatrix}$.

2. $\det B = -1$.

3. $B^{-1} = \frac{1}{-1}\begin{bmatrix} -2 & -3 & -1 \\ 1 & -1 & -1 \\ -3 & 1 & 2 \end{bmatrix}^T = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 1 & -1 \\ 1 & 1 & -2 \end{bmatrix}$.

$$C = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 0 \\ 1 & 2 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{bmatrix}.$$

1.The matrix of cofactors is $\begin{bmatrix} 0 & 0 & 0 & 24 \\ 0 & 0 & 32 & -24 \\ 0 & 48 & -32 & 0 \\ 24 & -12 & 0 & 0 \end{bmatrix}$.

2.$\det C = 96$.

3. $C^{-1} = \frac{1}{96}\begin{bmatrix} 0 & 0 & 0 & 24 \\ 0 & 0 & 32 & -24 \\ 0 & 48 & -32 & 0 \\ 24 & -12 & 0 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & 0 & 1/4 \\ 0 & 0 & 1/2 & -1/8 \\ 0 & 1/3 & -1/3 & 0 \\ 1/4 & -1/4 & 0 & 0 \end{bmatrix}$.

2. Find the determinant by rewriting the matrix in a triangular form

$$C = \begin{bmatrix} 1 & 1 & -3 & 4 & 1 \\ -3 & -1 & 11 & -11 & -3 \\ 1 & 2 & -3 & 7 & -2 \\ 2 & 1 & -6 & 8 & 3 \\ -2 & 0 & 6 & -7 & 0 \end{bmatrix}.$$

Solution: By performing the following EROs we rewrite the matrix in the upper triangular form. (Start as R2=R2+3R1, R3=R3-R1, R4=R4-2R1, R5=R5+2R1, switch R2 and R3,etc) Remembering that each switch changes the sign and each multiplication of a row by a number (like R4=2*R4) changes the value of det, and then using that the det of a triangular matrix is the product of its diagonal elements, we obtain:

$\det C = -28$.

3. For which values of a the matrix $\begin{bmatrix} 2 & 1 & 3 \\ 0 & a+1 & 0 \\ a & 2 & 4 \end{bmatrix}$ is not invertable?

Solution: A matrix A is not invertable if $\det A = 0$.

We find: $det \begin{bmatrix} 2 & 1 & 3 \\ 0 & a+1 & 0 \\ a & 2 & 4 \end{bmatrix} = (a+1)(8-3a) = 0.$

Thus for $a = -1$ and $a = 8/3$ the matrix is not invertable.

4. Let $\det A = 5$ and $\det B = 10$.

A) Find $\det(B^3 A^2 B^{-1} A^T B^T)$.

Solution: $\det(B^3 A^2 B^{-1} A^T B^T) = 10^3 \cdot 5^2 \cdot \frac{1}{10} \cdot 5 \cdot 10 = 125,000$

B) Given $\det(5A^{-1}B) = 6250$ find the size of the matrices.

Solution: Assume that A is $n \times n$-matrix. Then $5^n \cdot \frac{1}{5} \cdot 10 = 6250$. Solving for n we find $n = 5$.

5. True of False? Explain.

a) If a matrix is skew symmetrix ($A^T = -A$) then it is not invertable.

Solution: **False**. For example, $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is skew symmetric and invertable.

b) Every matrix such that $A^k = I$ for some $k \geq 1$ has determinant equal to one.

Solution: **False**. For example, $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ has determinant -1 and its square is the identity.

c) Elementary row operations do not change the determinant of the matrix. (in other words, if A is obtained from B by an ERO then $\det A = \det B$.)

Solution: **False**. For example, switching two rows changes the sign of the determinant.

d) There exist such matrices A, B, C for which $(ABC)^{-1} = A^{-1}B^{-1}C^{-1}$

Solution: **True**. This is not true in general, but as a special case such matrices do exist. For example, if $A = C$ and is invertable, then $A^{-1} = C^{-1}$ and we have

$$(ABC)^{-1} = C^{-1}B^{-1}A^{-1} = A^{-1}B^{-1}C^{-1}.$$