MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 5 **MATH 2050** ANSWERS.

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1. Find the inverse of each of the following matrices or explain why it is not possible.

$$A = \begin{bmatrix} 10 & 2 \\ 5 & 4 \end{bmatrix}, \qquad A^{-1} = \frac{1}{30}\begin{bmatrix} 4 & -2 \\ -5 & 10 \end{bmatrix}$$

$B = \begin{bmatrix} 10 & 2 \\ 5 & 1 \end{bmatrix}$ $\quad \det B = 0$ thus the inverse does not exist.

$$C = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 1 & -1 \\ 1 & 1 & -2 \end{bmatrix}, \qquad C^{-1} = \begin{bmatrix} 1 & -1 & 2 \\ -5 & 7 & -11 \\ -2 & 3 & -5 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & -1 & 2 \\ -5 & 7 & -11 \\ -2 & 3 & -5 \end{bmatrix}, \qquad D^{-1} = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 1 & -1 \\ 1 & 1 & -2 \end{bmatrix}$$

Noticed that $C^{-1} = D$ and $D^{-1} = C$. They are inverses of each other!

$F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, this is not a square matrix so the inverse does not exist.

$G = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 3 & 3 \end{bmatrix}$. G^{-1} does not exist. (we can't make the identity matrix in the first block.)

2. Solve the system of equations by writing it in the form $AX = B$ and finding A^{-1}. Check your answer.

(a) $\begin{cases} x - 7y = 11 \\ -x + 2y = -1 \end{cases}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = -\tfrac{1}{5}\begin{bmatrix} 2 & 7 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 11 \\ -1 \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$$

(b) $\begin{cases} 10x + 15y = 5 \\ 101x + 203y = 102 \end{cases}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \tfrac{1}{515}\begin{bmatrix} 203 & -15 \\ -101 & 10 \end{bmatrix}\begin{bmatrix} 5 \\ 102 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

(c) $\begin{cases} 2x - y + 3z = 3 \\ 3x + y - z = -2 \\ x + y - 2z = 0 \end{cases}$

Use the inverse matrix C^{-1} from problem 1 to find

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ -5 & 7 & -11 \\ -2 & 3 & -5 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ -29 \\ -12 \end{bmatrix}$$

(d) $\begin{cases} 2x - y + 3z = 0 \\ 3x + y - z = 11 \\ x + y - 2z = 7 \end{cases}$

Use the inverse matrix C^{-1} from problem 1 to find

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ -5 & 7 & -11 \\ -2 & 3 & -5 \end{bmatrix} \begin{bmatrix} 0 \\ 11 \\ 7 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}$$

3. Explain why each of the following matrices is elementary. Find its inverse in the easy way.

$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, This matrix is obtained from the identity matrix by switching rows 2 and 3. Its inverse is the same as the matrix is : $A^{-1} = A$.

$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}$,

This matrix is obtained from the identity matrix by multiplying row 4 by 10. Its inverse is : $B^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}$,

$C = \begin{bmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

This matrix is obtained from the identity matrix by replacing row 1 with row 1 plus 4 times row 3. (R1:= R1+4R3)

Its inverse is : $C^{-1} \begin{bmatrix} 1 & 0 & -4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

$D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -3 & 0 & 0 & 1 \end{bmatrix}$,

This matrix is obtained from the identity matrix by replacing row 4 with row 4 minus 3 times row 1. (R4:= R4-3R1)

Its inverse is : $D^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{bmatrix}$

4. True of False? Explain.

   a) If a matrix has an inverse then it must be a square matrix. **True**

   b) Every square matrix is invertable. **False**

   c) There exists a non-invertable elementary matrix. **False**

   d) If matrix A is invertable then the system $AX = B$ has a unique solution for any vector-column B. **True** (assuming the dimension is right).