MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 4 **MATH 2050** ANSWERS

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1. Find matrix A if

$$\begin{bmatrix} 5 & 1 \\ 0 & -2 \end{bmatrix} - 2A = \begin{bmatrix} -3 & 2 \\ -1 & 7 \end{bmatrix} + A^T.$$

Answer: Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then we have

$$3a = 8, \quad 2b + c = -1, \quad 2c + b = 1, \quad 3d = -9.$$

Solving the system we find the matrix $A = \begin{bmatrix} 8/3 & -1 \\ 1 & -3 \end{bmatrix}$.

2. Let A, B, C be symmetric matrices. Determine whether $A + B + C$ is symmetric. What about ABC?

Answer: We have $A = A^T$, $B = B^T$, $C = C^T$. Thus

$$(A + B + C)^T = A^T + B^T + C^T = (A + B + C).$$

Therefore matrix $A + B + C$ is symmetric.

$$(ABC)^T = C^T B^T A^T = CBA \neq ABC.$$

Therefore ABC in not symmetric in general. (It may be symmetric for some special choices of A,B,C.)

3. Consited matrices

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 2 \\ 1 & 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 2 & -3 \\ -1 & 0 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 2 & 0 \end{bmatrix}.$$

Find the following products if they are defined

$$AB, \quad AC, \quad CA, \quad AB^T, \quad A^T B, \quad A^2, \quad B^2, \quad C^2$$

Answer: The following products are undefined: AB, AC, $A^T B$, B^2, C^2. The rest of them are: $CA = \begin{bmatrix} 3 & 6 & 13 \end{bmatrix}$, $AB^T = \begin{bmatrix} -5 & 5 \\ -6 & 4 \\ 3 & -3 \end{bmatrix}$, $A^2 = \begin{bmatrix} 4 & 2 & 4 \\ 2 & 0 & -2 \\ 0 & 2 & 4 \end{bmatrix}$.

4. A square matrix P is called **idempotent** if $P^2 = P$. Show that if P is idempotent and I is a unit matrix then $(I-P)^2$ is also idempotent.

   Answer:

   $$(I-P)^2 = (I-P)(I-P) = I - 2P + P^2 = I - 2P + P = I - P.$$

   Therefore matrix $I - P$ is idempotent.

   Bonus: Show that $P + AP - PAP$ is idempotent given that P is idempotent, and A is any square matric of the same size as P.

   Answer:

   $(P+AP-PAP)^2 = P^2+AP^2-PAP^2+PAP+APAP-PAPAP-P^2AP-AP^2AP+PAP^2AP$

   Replacing P^2 with P and canceling similar terms we get

   $$(P + AP - PAP)^2 = P + AP - PAP.$$

   Therefore matrix $P + AP - PAP$ is idempotent.

5. Write the following system of linear equations in the form $AX = B$

   $5x_1-6x_2-2x_3-7=0,\quad 2x_1+17x_3+14=0,\quad 20x_1+x_2-x_4-10=0,\quad x_2+x_4-1=0,\quad x_1+x_5=0$

   namely, identify matrices A, X, B and their dimentions.

   Answer: $A = \begin{bmatrix} 5 & -6 & -2 & 0 & 0 \\ 2 & 0 & 17 & 0 & 0 \\ 20 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$, $B = \begin{bmatrix} 7 \\ -14 \\ 10 \\ 1 \\ 0 \end{bmatrix}$.

6. Given augmented matrix for a homogeneous system of linear equations find the basic solutions and write the parametric solution in the vector form

   $$\left[\begin{array}{cccccc|c} 1 & -2 & 3 & -4 & 5 & -1 & 0 \\ 0 & 0 & 0 & 5 & 7 & 1 & 0 \\ 0 & 0 & 0 & 0 & 4 & 12 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

   Answer: $X = q\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 32 \\ 0 \\ 0 \\ 4 \\ -3 \\ 1 \end{bmatrix}$, where q, s, t are any independently chosen numbers.

7. Compose a word problem whose solution leads to matrix multiplication.