MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 4 **MATH 2050 Sec.3** ANSWERS

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1. Consider augmented matrix for a homogeneous system of linear equations.

   1.1 Rewrite the problem in the matrix form $AX = B$ (identify matrices A, X and B, and their dimentions).

   1.2 Find the basic solutions and write the parametric solution in the **vector form**.

(a)

$$\left[\begin{array}{cccccc|c} 1 & 3 & -1 & 4 & 1 & 5 & 0 \\ 0 & 0 & 1 & 3 & 2 & 4 & 0 \\ 0 & 0 & 0 & 0 & 5 & 15 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Solution

1.1 Matrix form is $AX = B$, where

$$A = \begin{bmatrix} 1 & 3 & -1 & 4 & 1 & 5 \\ 0 & 0 & 1 & 3 & 2 & 4 \\ 0 & 0 & 0 & 0 & 5 & 15 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Here A is 3×6-matrix, X is 6×1-matrix (column-vector), and B is 3×1-matrix (column-vector).

1.2 The augmented matrix can be made in REF by deviding row 3 by 5. The rank of the matrix of coefficients is 3, thus paramentric solution has $6 - 3 = 3$ parameters. Let $x_6 = t$, $x_4 = s$, $x_2 = q$. Then the solution in the vector form is as follows:

$$X = q \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -7 \\ 0 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \\ -3 \\ 1 \end{bmatrix}.$$

Basic vectors-solutions are:

$$\begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} -7 \\ 0 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \\ -3 \\ 1 \end{bmatrix}.$$

(b)

$$\left[\begin{array}{ccccc|c} 1 & 3 & -1 & 4 & 1 & 0 \\ 2 & 6 & -2 & 3 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Solution: 1.1 Matrix form is $AX = B$, where

$$A = \begin{bmatrix} 1 & 3 & -1 & 4 & 1 \\ 2 & 6 & -2 & 3 & 2 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Here A is 2×5-matrix, X is 5×1-matrix (column-vector), and B is 2×1-matrix (column-vector).

1.2 The augmented matrix can be made in REF by subtracting from row-2 two of row-1 ($R2 := R2 - 2R1$). The rank of the matrix of coefficients is 2, thus paramentric solution has $5 - 2 = 3$ parameters. From the second equation in REF it follow that $x_4 = 0$

Let $x_5 = t$, $x_3 = s$, $x_2 = q$. Then the solution in the vector form is as follows:

$$X = q \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

Basic vectors-solutions are:

$$\begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad s \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

2. Write the system of linear equations $\begin{cases} 5x_2 - 2x_6 - 1 = 0 \\ 2x_1 + 17x_4 + 10 = 0 \\ 20x_3 + x_5 - 6 = 0 \end{cases}$ in the form $AX = B$, namely, identify matrices A, X, B and their dimentions. Find the parametric solution and write it in the **vector form**.

Solution: Matrix form is $AX = B$, where

$$A = \begin{bmatrix} 0 & 5 & 0 & 0 & 0 & -2 \\ 2 & 0 & 0 & 17 & 0 & 0 \\ 0 & 0 & 20 & 0 & 1 & 0 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ -10 \\ 6 \end{bmatrix}$$

Here A is 3×6-matrix, X is 6×1-matrix (column-vector), and B is 3×1-matrix (column-vector).

We can find parametric solution of the system:

$$x_6 = t, \quad x_4 = s, \quad x_3 = q, \quad x_5 = 6 - 20q, x_2 = \frac{1+2t}{5}, \quad x_1 = -\frac{10+17s}{2}.$$

Then the solution is the vector form is:

$$X = \begin{bmatrix} -5 \\ 0.2 \\ 0 \\ 0 \\ 6 \\ 0 \end{bmatrix} + s \begin{bmatrix} 17/2 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 0.4 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + q \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -20 \\ 0 \end{bmatrix}.$$

3. Find matrix A if

$$3\begin{bmatrix} 1 & 1 & 0 \\ 1 & 4 & -2 \end{bmatrix}^T + 2A = \begin{bmatrix} -2 & 1 \\ -1 & 7 \\ 0 & 5 \end{bmatrix}.$$

Answer: $A = \begin{bmatrix} -2.5 & -1 \\ -2 & -2.5 \\ 0 & 5.5 \end{bmatrix}$

4. Give a **definition** and an **example** of each:

(a) diagonal matrix,

Answer: only main diagonal may contain non-zero element.

(b) non-square matrix,

Answer: number of rows differs from number of columns.

(c) symmetric matrix,

Answer: does not change after transposition $A^T = A$.

(d) skew-symmetric matrix,

Answer: changes the sign after transposition $A^T = -A$.

(e) idempotent matrix,

Answer: does not change after multiplication by itself: $AA = A$.

(f) identity marix.

Answer: a square matrix with all 1 on the main diagonal.

5. Let A, B be **symmetric** matrices, and C, D **skew-symmetric** matrices. Determine if the following is symmetric, skew symmetric or neither:

$$A + B, \quad A + C, \quad D + C, \quad A + B + C, \quad AB, \quad AC, \quad CD, \quad C + C^T.$$

Answer:

$A + B$ is symmetric;

$A + C$ is neither;

$D + C$ is skew-symmetric;

$A + B + C$ is neither;

AB is neither;

AC is neither;

CD is neither;

$C + C^T$ is BOTH (because the result is zero matrix);

6. Consider matrices

$$A = \begin{bmatrix} 0 & 2 & 3 \\ 1 & 0 & 2 \\ 1 & 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 2 & -3 \\ -1 & 0 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix}.$$

Find the following products if they are defined

$$BA, \quad AC^T A, \quad A^T B^T, \quad A^T B, \quad A^2, \quad B^2, \quad C^2$$

Answers: $BA = \begin{bmatrix} -1 & 0 & 7 \\ 2 & -2 & -5 \end{bmatrix}$, $A^T B^T = (BA)^T = \begin{bmatrix} -1 & 2 \\ 0 & -2 \\ 7 & -5 \end{bmatrix}$,

$A^2 = AA = \begin{bmatrix} 5 & 0 & 1 \\ 2 & 2 & 1 \\ -1 & 2 & 4 \end{bmatrix}$.

Other combinations are undefined.

7. Compose a word problem whose solution leads to matrix multiplication.