# MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

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ASSIGNMENT 4 **MATH 2050 sect. 3** ANSWERS

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1. Find matrix $A$ if

$$4A - \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -1 & 7 \end{bmatrix} - 2A.$$

*Answer*

$$6A = \begin{bmatrix} 0 & 4 \\ -2 & 7 \end{bmatrix} \quad \rightarrow \quad A = \begin{bmatrix} 0 & 2/3 \\ -1/3 & 7/6 \end{bmatrix}.$$

2. A square matrix $B$ is called **skew-symmetric** if $B^T = -B$. Let $A$ be a square matrix. Show that $B = A - A^T$ is skew-symmetric.

*Solution:* Calculate $B^T = (A - A^T)^T = A^T - (A^T)^T = A^T - A = -(A - A^T) = -B$. Thus, by dfinition, $B$ is skew-symmetric.

3. Consited matrices

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 2 & 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 2 & -3 & 1 & 5 \\ -1 & 0 & 2 & 1 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}.$$

Find the following products if they are defined

$$AB, \quad AC, \quad CA, \quad AB^T, \quad A^TB, \quad A^2, \quad B^2, \quad C^2$$

*Answer* $AB$, $AC$, $A^2$, $B^2$ are undefined.

$$CA = \begin{bmatrix} 3 & 6 & 13 & 14 & 23 \\ -1 & -2 & -3 & -4 & -5 \end{bmatrix}, \quad AB^T = \begin{bmatrix} 24 & 29 \\ 15 & 21 \end{bmatrix}, \quad C^2 = \begin{bmatrix} 7 & 6 \\ -3 & -2 \end{bmatrix},$$

$$A^TB = \begin{bmatrix} 0 & 2 & -3 & 1 & 5 \\ 0 & 4 & -6 & 2 & 10 \\ -2 & 6 & -5 & 5 & 23 \\ -1 & 8 & -10 & 5 & 24 \\ -4 & 10 & -7 & 9 & 41 \end{bmatrix}$$

4. The **trace** of a square matrix $A$, denoted tr$A$, is the sum of the elements on the main diagonal of $A$. Show that if $A$ and $B$ are $n \times n$ matrices then

$$\mathrm{trA^T = trA, \quad tr(A+B) = trA + trB, \quad tr(AB) = tr(BA),}$$

*Solution*

1. Note that after transposition the main diagonal of a matrix remains the same. Thus, the sum of the elements on the main diagonal is the same for both $A$ and $A^T$.

2. By definition, $(A+B)_{ij} = A_{ij} + B_{ij}$ for all $i, j = 1, 2, ...n$. In particular, for the diagonal elements we have $(A+B)_{jj} = A_{jj} + B_{jj}$ for all $j = 1, 2, ...n$. Thus

$$\mathrm{tr(A+B)} = \sum_{\mathrm{j=1}}^{\mathrm{n}} (\mathrm{A_{jj} + B_{jj}}) = \sum_{\mathrm{j=1}}^{\mathrm{n}} \mathrm{A_{jj}} + \sum_{\mathrm{j=1}}^{\mathrm{n}} \mathrm{B_{jj}} = \mathrm{trA + trB}.$$

3. Let at first $n = 3$, so we are dialing with $3 \times 3$ matrices $A$ and $B$. Introduce $C = AB$ and $D = BA$. Then the diagonal elements of $C$ and $D$ are

$$C_{jj} = A_{j1}B_{1j} + A_{j2}B_{2j} + A_{j3}B_{3j} = \sum_{i=1}^{3} A_{ji}B_{ij}, \quad D_{jj} = \sum_{i=1}^{3} B_{ji}A_{ij}, \quad j = 1, 2, 3.$$

Then

$$\mathrm{trAB} = \mathrm{trC} = \mathrm{C}_{11} + \mathrm{C}_{22} + \mathrm{C}_{33} = \sum_{\mathrm{j}=1}^{3} \mathrm{C}_{\mathrm{jj}} = \sum_{\mathrm{j}=1}^{3}\sum_{\mathrm{i}=1}^{3} \mathrm{A}_{\mathrm{ji}}\mathrm{B}_{\mathrm{ij}}.$$

and

$$\mathrm{trBA} = \mathrm{trD} = \sum_{\mathrm{j}=1}^{3} \mathrm{D}_{\mathrm{jj}} = \sum_{\mathrm{j}=1}^{3}\sum_{\mathrm{i}=1}^{3} \mathrm{B}_{\mathrm{ji}}\mathrm{A}_{\mathrm{ij}}.$$

Each sum above contains 9 terms, and if you compare them, you find that they are the same:

$$A_{11}B_{11} + A_{12}B_{21} + A_{13}B_{31} + A_{21}B_{12} + A_{22}B_{22} + A_{23}B_{32} + A_{31}B_{13} + A_{32}B_{23} + A_{33}B_{33}.$$

Thus we have it proven for $n = 3$.

With a little harder work, the same can be proven for arbitrary $n$.

5. Write the following system of linear equations in the form $AX = B$

$$x_1 - x_2 + 3x_3 = 4, \quad x_2 + 10x_3 = -4, \quad 20x_1 - x_3 = 0 \quad , x_4 = 1,$$

namely, identify matrices $A, X, B$ and their dimentions.

*Answer*

$$A = \begin{bmatrix} 1 & -1 & 3 & 0 \\ 0 & 1 & 10 & 0 \\ 20 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad B = \begin{bmatrix} 4 \\ -4 \\ 0 \\ 1 \end{bmatrix},$$

6. Given agmented matrix of coefficients of a homogeneous system find the basic solutions and write the parametric solution in the vector form

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 1 \\ 0 & 0 & 2 & -6 & 4 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

*Answer*:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} t + \begin{bmatrix} -13 \\ 0 \\ 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} q + \begin{bmatrix} 1 \\ 0 \\ -2 \\ 0 \\ 1 \\ 0 \end{bmatrix} r + \begin{bmatrix} 2 \\ 0 \\ -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} s.$$

7. Compose a word problem whose solution leads to matrix multiplication.