MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Assignment 3 **MATH 2050 sect. 3** Answers

1. For each system of linear equations given below identify the rank of the matrix of coefficients. Find the solution (if it exists). In each solvable case observe the relation between the rank, number of parameters and total number of unknowns.

(a) $\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 2 & 1 & 4 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 1 & 1/2 & 2 \end{array}\right]$ $(R_2 := R_2/2)$

Answer Rank =2; number of parameters =2; number of unknowns=4;

$$x_4 = t, \quad x_3 = 2 - t/2, \quad x_2 = s, \quad x_1 = -1 - 2s - 5t/2.$$

(b) $\left[\begin{array}{cccc|c} 0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 2 & 3 & 10 \end{array}\right] \to \left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$

$(R_1 := R_2,\ R_2 := R_3,\ R_3 := R_1,\ R_2 := R_2 - 3R_3,\ R_2 := R_2/2\ )$

Answer Rank =3; number of parameters =1; number of unknowns=4;

$$x_1 = t, \quad x_2 = 3, \quad x_3 = 2, \quad x_4 = 2.$$

(c) $\left[\begin{array}{cccc|c} 2 & 2 & 2 & 2 & 3 \\ 4 & 4 & 4 & 4 & 3 \\ 1 & 1 & 1 & 1 & 3 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & -9 \\ 0 & 0 & 0 & 0 & -3 \end{array}\right]$

$(R_1 <-> R_3,\ R_2 := R_2 - 4R_1,\ R_3 := R_3 - 2R_1,$

Answer Rank =1; number of unknowns=4;

There is no solutions to this system (thus, no parameters).

(d) $\left[\begin{array}{cccc|c} 2 & 2 & 2 & 2 & 4 \\ 4 & 4 & 4 & 4 & 8 \\ 1 & 1 & 1 & 1 & 2 \end{array}\right] \to \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$(R_1 <-> R_3,\ R_2 := R_2 - 4R_1,\ R_3 := R_3 - 2R_1,$

Answer Rank =1; number of parameters =3; number of unknowns=4;

$$x_1 = 2 - t - s - r, \quad x_2 = t, \quad x_3 = s, \quad x_4 = r.$$

2. Find an equation of the curve or surface passing through given points:

(a) line $ax + by = c$ through points $(1, 2)$ and $(-100, 200)$;

Solution

$(1, 2) \to a + 2b - c = 0$

$(-100, 200) \to -100a + 200b - c = 0$

Solve the system to obtain $c = t, b = 101t/400, a = 198t/400$. Substitute back to the equation to get $198x + 101y = 400$.

(b) plane $ax + by + cz + d = 0$ through $(0, 1, 20)$, $(1, 20, 0)$, $(1, 0, 0)$

Solution

$(0, 1, 20) \to b + 20c + d = 0$

$(1, 20, 0) \to a + 20b + d = 0$

$(1, 0, 0) \to a + d = 0$

Solve the system to obtain $d = t, c = -t/20, b = 0, a = -t$. Substitute back to the equation to get $20x + z = 20$.

(c) circle $x^2 + y^2 + ax + by + c = 0$ through $(-2, 1)$, $(5, 0)$, $(4, 1)$.

Solution

$(-2, 1) \to -2a + b + c = -5$

$(5, 0) \to 5a + c = -25$

$(4, 1) \to 4a + b + c = -17$

Solve the system to obtain $a = -2, b = 6, c = -15$. Substitute back to the equation to get $x^2 + y^2 - 2x + 6y - 15 = 0$.

(d) parabola $x^2 + ax + by + c = 0$ through $(0, 9)$, $(4, 1)$, $(2, 3)$.

Solution

$(0, 9) \to 9b + c = 0$

$(4, 1) \to 4a + b + c = -16$

$(2, 3) \to 2a + 3b + c = -4$

Solve the system to obtain $a = -2, b = 6, c = -15$. Substitute back to the equation to get $x^2 - 8x - 2y + 18 = 0$

3. Find value a such that the system has non-trivial solutions. Find the solutions.

(a) $\begin{cases} x - 2y + z = 0 \\ x + ay - 3z = 0 \\ -x + 6y - 5z = 0 \end{cases}$

Solution: $\left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 1 & a & -3 & 0 \\ -1 & 6 & -5 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & a-2 & 0 \end{array}\right]$

$(R_3 <-> R_2, R_2 := R_2 + R_1,\ R_2 := R_2/4,\ R_3 := R_3 - R_1,\ R_3 := R_3 - (a+2)R_2)$

From REF we observe that a non-trivial solution will occure only in case $a - 2 = 0$.

For $a = 2$ we have $\left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$, thus parametric solution has the form $x = y = z = t$, which gives a non-trivial solution for any choice of parameter $t \neq 0$, for example, $x = y = z = 2006$.

(b) $\begin{cases} x + 2y + z = 0 \\ x + 3y + 6z = 0 \\ 2x + 3y + az = 0 \end{cases}$

Solution: $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 1 & 3 & 6 & 0 \\ 2 & 3 & a & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & a+3 & 0 \end{array}\right]$

$(R_2 := R_2 - R_1,\ R_3 := R_3 - 2R_1,\ R_3 := R_3 + R_2)$

From REF we observe that a non-trivial solution will occure only in case $a + 2 = 0$.

For $a = -3$ we have $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$, thus parametric solution has the form $z = t$, $y = -5t$, $x = 9t$, which gives a non-trivial solution for any choice of parameter $t \neq 0$, for example, $x = 9, y = -5, z = 1$.

4. True or False?

   (a) Every system of linear equations has a solution. **False**

   (b) Every homogeneous system of linear equations has a non-trivial solution. **False**

   (c) If a system of linear equations has zero solution then it must be homogeneous. **True**

   (d) If in a system of linear equations the number of variables is less then the number of equations then the system has no solutions. **False**

5. Compose your own True-or-False question. Answer is.