MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 2 **MATH 2050 Sec 3** ANSWERS

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1. There are geese and pigs in a farm. They collectively have 20 legs. How many geese and pigs are in the farm? Is the answer unique? How many answers are possible? How do you know that you listed them all?

   Answer: We need to solve equation $2x + 4y = 20$. Here x, y are non-negative integers. Thus all the six possibilities are: 10 geese and no pigs; 8 geese and one pig; 6 geese and two pigs; 4 geese and 3 pigs; 2 geese and 4 pigs; no geese and 5 pigs;

2. Give an example of an augmented matrix of a system of linear equations with its matrix of coefficients in the row-echelon form (REF) such that:

   a) the system has a unique solution;

   Answer: $\left[\begin{array}{ccr|c} 1 & 2 & -3 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$

   b) the system has infinitely many solutions;

   Answer: $\left[\begin{array}{ccr|c} 1 & 2 & -3 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

   c) the system has no solutions;

   Answer: $\left[\begin{array}{ccr|c} 1 & 2 & -3 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 3 \end{array}\right]$

3. Solve the given systems by reduction corresponding Augmented Matrix to Row-Echelon Form (REF). Find the rank of the matrix of coefficients.

   (a) $\begin{cases} 2x + 3y + z = 1 \\ x + y + z = 3 \\ 3x + 4y + 2z = 4 \end{cases}$

   Solution: the system in REF is $\left[\begin{array}{ccr|r} 1 & 1 & 1 & 3 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 0 & 0 \end{array}\right]$

   Rank is 2. Answer: $x = 8 - 2t$, $y = t - 5$, $z = t$, where t is any number.

   (b) $\begin{cases} x - y + 2z = 4 \\ 2x + 3y - z = 1 \\ 7x + 3y + 4z = 7 \end{cases}$

   Answer: the system is inconsistent. Rank is 2.

(c) $\begin{cases} -x+2y-z=2 \\ -2x+2y+z=4 \\ 3x+2y+2z=5 \\ -3x+8y+5z=17 \end{cases}$

Answer: $x=0$, $y=1.5$, $z=1$; The rank is 3.

4. In each of the following find conditions for a, b, c such that the system has no solutions, a unique solution, or infinitely many solutions.

(a) $\begin{cases} x-y+2z=a \\ 2x+3y-z=b \\ 7x+3y+4z=c \end{cases}$

Solution: the system in the REF becomes: $\left[\begin{array}{rrr|r} 1 & -1 & 2 & a \\ 0 & 1 & -1 & (b-2a)/5 \\ 0 & 0 & 0 & c-2b-3a \end{array}\right]$

Thus the system is inconsistent when $c \neq 2b+3a$. Otherwise it has infinitely many solutions.

(b) $\begin{cases} x-y+2z=1 \\ 2x+3y-z=2 \\ 7x+3y+az=1 \end{cases}$

Solution: the system in the REF becomes: $\left[\begin{array}{rrr|r} 1 & -1 & 2 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & a-4 & -6 \end{array}\right]$

Thus the system is inconsistent when $a=4$. Otherwise it has a unique solution.

(c) $\begin{cases} x-y+2z=1 \\ 2x+3y-z=1 \\ 7x+3y+bz=5 \end{cases}$

Solution: the system in the REF becomes: $\left[\begin{array}{rrr|r} 1 & -1 & 2 & 1 \\ 0 & 1 & -1 & -1/5 \\ 0 & 0 & b-4 & 0 \end{array}\right]$

Thus the system has infinitely many solutions when $b=4$. Otherwise it has a unique solution.

5. Find all solutions to the following system in parametric form in two ways. Use sample value of parameter to obtain a particular numeric solution from one of the forms. Then find value of parameter in another form that yield the same numeric solution.

$\begin{cases} x+2y-3z=4 \\ 3x-2y=6 \end{cases}$

Answer:

Way 1. From the second equation find $x=2+2y/3$. Take $y=3t$, where t is any number. Then parametric solution is $x=2+2t$, $y=3t$, $z=(8t-2)/3$, t is any number.

Way 2. From the second equation find $y = 3x/2 - 3$. Take $x = 2s$, where s is any number. Then parametric solution is $x = 2s$, $y = 3s - 3$, $z = (8s - 10)/3$, s is any number.

Take $t = 0$ in the first form of the solution. It gives particular numeric solution $x = 2, y = 0, z = -2/3$. This solution also corresponds to value $s = 1$ in the second form.