MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 1 **MATH 2050 Sec. 3** DUE: SEPT 14, 2007

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1. Find all solutions for the following systems of linear equations by writing the solution in parametric form. How many parameters are in the solution?

(a) $\begin{cases} x+y-z=2 \\ x-y=0 \end{cases}$

Solution: From the second equation we find that $x=y$. Thus if x is chosen arbitrary, say $x=t$ then $y=t$ as well. Substitute this values in the first equation to get $2t-z=2$. Thus $z=2t-2$. Finally, we write the answer in the form $\begin{cases} x=t \\ y=t \\ z=2t-2, \end{cases}$

where t is any number.

There is one parameter, t.

(b) $\begin{cases} x+2y-3z=4 \\ x-2y=1 \end{cases}$

Solution: From the second equation we find that $x=2y+1$. Thus if y is chosen arbitrary, say $y=t$ then $x=2t+1$. Substitute this values in the first equation to get $(2t+1)+2t-3z=4$. Thus $3z=4t-3$. Finally, we write the answer in the form

$\begin{cases} x=2t+1 \\ y=t \\ z=(4t-3)/3, \end{cases}$ where t is any number.

There is one parameter, t.

(c) $\begin{cases} x+2y-3z+u=4 \\ x+z=0 \end{cases}$

Solution: From the second equation we find that $x=-z$. Thus if z is chosen arbitrary, say $z=t$ then $x=-t$. Substitute this values in the first equation to get $-t+2y-3t+u=4$. We still have freedom to choose, or example, $y=s$. Then $-4t+2s+u=4$. Thus $u=4+4t-2s$. Finally, we write the answer in the form $\begin{cases} x=-t \\ y=s \\ z=t \\ u=4+4t-2s, \end{cases}$

where t and s are any numbers.

There are two parameters, t and s.

(d) $x+y+z+u+v+10=0$.

Solution: We can arbitrary choose values of four variables and the remaining variables will be defined by them. For example, $\begin{cases} x=t \\ y=s \\ z=q \\ u=r \\ v=-10-t-s-q-r, \end{cases}$ where t,s,q,r are any numbers.

There are four parameters, t,s,q,r.

2. Solve each of the systems algebraically and geometrically (or argue that it does not have a solution). Write the augmented matrix corresponding to each of the systems.

(a) $\begin{cases} x+2y=1 \\ x+1=0 \\ y-1=0 \end{cases}$

Answer: Algebraic solution gives $x=-1$, $y=1$.

Geometric solution comes from graphing three lines: $y=(1-x)/2$, $x=-1$, $y=1$, and observing that they all intersect at point $(-1,1)$.

Augmented matrix is $\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{array}\right]$

(b) $\begin{cases} x+2y=3 \\ 10y+5x=30 \end{cases}$

Answer:

Algebraic solution leads to the conclution that the system is inconsistent (no such pair (x,y) is possible).

Geometric solution comes from graphing two lines: $y=(3-x)/2=-x/2+3/2$, $y=-x/2+3$, and observing that they are parallel, and thus there is no point of intersection.

Augmented matrix is $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 5 & 10 & 30 \end{array}\right]$

(c) $\begin{cases} x+2y=1 \\ x-1=0 \\ y+1=0 \end{cases}$

Answer:

Algebraic solution leads to the conclution that the system is inconsistent (no such pair (x,y) is possible).

Geometric solution comes from graphing three lines: $y=(1-x)/2$, $x=1$, $y=-1$, and observing that there is no common point of intersection.

Augmented matrix is $\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{array}\right]$

(d) $\begin{cases} x+2y=1 \\ x-2y=1 \end{cases}$

Answer:

Algebraic solution gives $x=1$, $y=0$.

Geometric solution comes from graphing two lines: $y=(1-x)/2$, $y=(x-1)/2$, and observing that they intersect at point $(1,0)$.

Augmented matrix is $\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 1 & -2 & 1 \end{array}\right]$

3. Write a linear system corresponding to the given augmented matrix.

$$\left[\begin{array}{ccccc|c} 6 & 2 & -3 & 4 & 1 & 0 \\ 5 & 0 & 0 & 1 & 200 & 2 \end{array}\right]$$

Answer:

$$\begin{cases} 6x + 2y - 3z + 4u + 5v = 0 \\ 5x + u + 200v = 2 \end{cases}$$

4. Give an example of a system of three linear equations in two variables that has infinitely many solutions.

Answer:

$$\begin{cases} x + y = 1 \\ 2x + 2y = 2 \\ 3x + 3y = 3 \end{cases}$$

such a system has parametric solution $\begin{cases} x = t \\ y = 1 - t, \end{cases}$ where t is arbitrary number. This gives infinitly many solutions.

5. In order to cook a *party-style* pizza Margo needs 10 mushrooms and 3 large tomatoes. For a *casual-style* pizza she needs 5 mushrooms and 2 large tomatoes. Given that 200 mushrooms and 70 large tomatoes were consumed in a cooking, find how many pizzas of each style were cooked by Margo.

Solution: Let x be the number of party-style pizzas and y be the number of casual-style pizzas. Then we have system of linear equations $\begin{cases} 10x + 5y = 200 \\ 3x + 2y = 70 \end{cases}$ Solving the system we have $x = 10, y = 20$.

Thus the answer is: Margo cooked 10 party-style pizzas and 20 casual-style pizzas, and thus she must have been exhausted!

6. Compose your own word problem that requires solution of a system of linear equations. Solve the problem.