MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 1 **MATH 2050-003** ANSWERS

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1. Find all solutions for the following problems by writing the solution in parametric form

   (a) $20x - y = 10$

   Answer: Note that there are two ways to write answer to this questions. One way to write the answer is: $x = t$, $y = 20t - 10$, where t is any real number.

   (b) $10x + 3y + 5z = 6$

   Answer: $z = t$, $y = s$, $x = (-3s - 5t)/10$, where t, s are any real numbers.

   (c) $ax + by + cz + dw = e$, where a, b, c, d are nonzero constants.

   Answer: $w = r$, $z = t$, $y = s$, $x = (e - bs - ct - dr)/a$, where r, t, s are any real numbers.

2. Solve each of the systems algebraically and geometrically (or argue that it does not have a solution). Write the augmented matrix corresponding to each of the systems.

   (a) $x + y + 4 = 0$

   $9x - 3y = 0$

   Solution.

   Algebraic: Substutite $x = -4 - y$, found from the first equation, into the second equation to get $-9(4 + y) - 3y = 0$; thus $-12y = 36$; $y = -3$. So, $x = -1$. Answer: $x = -1; y = -3$.

   Geometric solution: graph of the first equation is a line $y = -4 - x$ with slope -1 and y-intersept at $(0, -4)$; graph of the second equation is a line $y = 3x$ with slope 3 passing through the origin. The two lines intersect at point $(-1, -3)$. Thus the answer is $x = -1; y = -3$.

   (b) $2x + y = 3$

   $3x - y = 2$

   $20x - 30y = -10$

   Answer. Algebraic: $x = 1, y = 1$ sutisfies all three equations.

   Geometric: Graph of each equation is a line. All three lines intersect at point $(1, 1)$.

   (c) $2x + y = 3$

   $3x - y = 2$

   $20x - 30y = -1$

   Answer. Algebraic: There is no such number x and y which satisfy all three equations. Thus there is no solution.

   Geometric: Graph of each equation is a line, but there is no an intersection point common to all three lines. Thus there is no solution.

(d) $2x + y = 3$

$2y + 4x = 6$

Answer. Algebraic: The second equation is obtained from the first by multiplication of every number by 2. Thus the two equations have the same set of solutions. The solution in the parametric form can be written as $x = t$, $y = 3 - 2t$, where t is any number.

Geometric: Graphs of both equations give the same line, $y = 3 - 2x$. All point on that line are solutions. Each particular t in the parametric solution gives one point from this line. For example, $t = 0$ gives point $(0, 3)$. Equivalently, $x = 0$, $y = 3$ is one of the infinitly many solutions to the system.

3. Write a linear system corresponding to the given augmented matrix.

(a) $\left[\begin{array}{cc|c} 4 & 12 & 16 \\ 3 & -9 & -1 \end{array}\right]$

Answer. $\begin{cases} 4x + 12y = 16 \\ 3x - 9y = -1 \end{cases}$

(b) $\left[\begin{array}{rrrr|r} -1 & 2 & -3 & 4 & 5 \\ 0 & -10 & 0 & 1 & 100 \end{array}\right]$

Answer.

4. Margo needs 42mg of vitamin A and 65mg of vitamin D per day. She has two supplements: the first contains 10% vitamin A and 25% vitamin D; the second contains 20% vitamin A and 25% vitamin D. How much of each supplement should she eat each day?

Solution. Suppose that Margo eats x mg of supplement one and y mg of supplement two per day. Then she gets $(0.1x + 0.2y)$ mg of vitamin A and $(0.25x + 0.25y)$ mg of vitamin D. Since Margo needs 42 mg of A and 65mg of D, we have a system of linear equations for x and y:

$$\begin{cases} 0.1x + 0.2y = 42 \\ 0.25x + 0.25y = 65 \end{cases}$$

Solving the system we find $x = 100$ and $y = 160$.

Answer. Margo should eat 100mg of supplement one and 160mg of supplement two per day.

5. Compose your own word problem that requires solution of a system of linear equations. Solve the problem.