Assignment No. 8 **MATHEMATICS 2050** Fall 2004

Due as follows:

Dr. Kondratieva	Tuesday November 9	in class or assignment box #47
Dr. Goodaire	Wednesday November 10	10:00 a.m.
Dr. Yuan	Wednesday November 10	in class

[4] 1. Find a condition on a, b, and c in order that the system $\begin{aligned} x + y - z &= a \\ 2x - 3y + 5z &= b \\ 5x \quad\;\; + 2z &= c \end{aligned}$ have a solution. If there is a solution, is this unique? Explain.

[3] 2. Let $\mathsf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\mathsf{v}_2 = \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$, $\mathsf{v}_3 = \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}$, $\mathsf{v}_4 = \begin{bmatrix} 1 \\ 1 \\ 13 \end{bmatrix}$. Determine whether these vectors are linearly independent or linearly dependent. If they are linearly dependent, write down a nontrivial linear combination of the vectors which equals the zero vector.

[1] 3. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 6 & 8 & 9 \end{bmatrix}$. Write down an elementary matrix E so that $EA = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 6 & 8 & 9 \end{bmatrix}$.

[1] 4. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix}$. Find A^{-1} without calculation and explain your reasoning.

[3] 5. Find an LU and an LDU factorization of $A = \begin{bmatrix} -2 & 4 & 6 \\ 1 & 2 & -8 \\ 4 & 0 & -5 \end{bmatrix}$.

6. Let $A = \begin{bmatrix} 1 & 6 & 7 \\ 5 & 25 & 36 \\ -2 & -27 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ -2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 6 & 7 \\ 0 & -5 & 1 \\ 0 & 0 & 7 \end{bmatrix} = LU$.

[3] (a) Use the given factorization of A to solve the linear system $A\mathsf{x} = \mathsf{b}$, where $\mathsf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

[1] (b) Express b as a linear combination of the columns of A.

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