Assignment 2 Solutions

Problem 1: (Section 1.4 Question 2) Simplify each of the following statements.

(c.)$[(p \to q) \vee (q \to r)] \wedge (r \to s)$

Solution:
(c.)

$$\iff [(\neg p \vee q) \vee (\neg q \vee r)] \wedge (r \to s)$$
$$\iff [\neg p \vee q \vee \neg q \vee r] \wedge (r \to s)$$
$$\iff [(\neg p \vee r) \vee (\neg q \vee q)] \wedge (r \to s)]$$
$$\iff [(\neg p \vee r) \vee 1] \wedge (r \to s)$$
$$\iff 1 \wedge (r \to s)$$
$$\iff r \to s$$
$$\iff \neg r \vee s$$

The above is determined using the following rules:

$$p \to q \iff \neg p \vee q$$
$$\neg q \vee q \iff 1$$
$$1 \wedge q \iff q$$

Problem 2: (Section 1.4 Question 3) Using truth tables, verify the following absorption properties.

(b.) $(p \wedge (p \vee q)) \iff p$

Solution:
(b.)

p	q	$p \vee q$	$p \wedge (p \vee q)$	$p \leftrightarrow (p \wedge (p \vee q))$
T	T	T	T	T
T	F	T	T	T
F	T	T	F	T
F	F	F	F	T

$p \leftrightarrow (p \wedge (p \vee q))$ is a tautology, since always true, therefore, $p \iff p \wedge (p \vee q)$

Problem 3: (Section 1.4 Question 4) Using the properties in the text together with the absorption properties given in Exercise 3, establish each of the following logical equivalences.

(b.) $[p \to (q \to r)] \iff [(p \wedge (\neg r)) \to (\neg q)]$

(c.) $[\neg(p \leftrightarrow q)] \iff [p \leftrightarrow (\neg q)]$

Solution:

The idea in this question is to work out left side and right side separately, trying to match the results.

(b.) Left Hand Side

$$\iff p \rightarrow (\neg q \vee r)$$
$$\iff \neg p \vee (\neg q \vee r)$$

Right Hand Side

$$\iff \neg(p \wedge \neg r) \vee (\neg q)$$
$$\iff (\neg p \vee r) \vee (\neg q)$$
$$\iff \neg p \vee (r \vee \neg q)$$
$$\iff \neg p \vee (\neg q \vee r)$$

Now the left hand side is equal to the right hand side. $\neg p \vee (\neg q \vee r) \iff \neg p \vee (\neg q \vee r)$

Therefore, $[p \rightarrow (q \rightarrow r)] \iff [(p \wedge (\neg r)) \rightarrow (\neg q)]$

(c.) Left Hand Side

$$\iff \neg[(p \rightarrow q) \wedge (q \rightarrow p)]$$
$$\iff \neg[(\neg p \vee q) \wedge (\neg q \vee p)]$$
$$\iff (p \wedge \neg q) \vee (q \wedge \neg p)$$

Right Hand Side

$$\iff (p \rightarrow \neg q) \wedge (\neg q \rightarrow p)$$
$$\iff (\neg p \vee \neg q) \wedge (q \vee p)$$
$$\iff ((\neg p \vee \neg q) \wedge q) \vee ((\neg p \vee \neg q) \wedge p)$$
$$\iff (\neg p \wedge q) \vee (\neg q \wedge q) \vee (\neg p \wedge p) \vee (\neg q \wedge p)$$
$$\iff (\neg p \wedge q) \vee 0 \vee 0 \vee (\neg q \wedge p)$$
$$\iff (\neg p \wedge q) \vee (\neg q \wedge p)$$
$$\iff (p \wedge \neg q) \vee (q \wedge \neg p)$$

Now the left hand side is equal to the right hand side. $(p \wedge \neg q) \vee (q \wedge \neg p) \iff (p \wedge \neg q) \vee (q \wedge \neg p)$

Therefore, $[\neg(p \leftrightarrow q) \iff p \leftrightarrow (\neg q)]$

Problem 4: (Section 1.4 Question 8) Express each of the following statements in disjunctive normal form.

(c.) $p \rightarrow q$
(d.) $(p \rightarrow q) \wedge (q \wedge r)$

Solution:
(c.)

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Therefore the disjunction normal form is: $(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$.
(d.)

p	q	r	$p \rightarrow q$	$q \wedge r$	$(p \rightarrow q) \wedge (q \wedge r)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	F	F
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	F	F
F	F	F	T	F	F

Therefore the disjunction normal form is: $(p \wedge q \wedge r) \vee (\neg p \wedge q \wedge r)$.

Problem 5: (Review Exercises Chapter 1 Question 14) Determine whether or not each of the following arguments is valid.

(a.)

$$\neg((\neg p) \wedge q)$$

$$\neg(p \wedge r)$$

$$\underline{r \vee s}$$

$$q \rightarrow s$$

(b.)

$$p \vee (\neg q)$$

$$(t \vee s) \rightarrow (p \vee r)$$

$$(\neg r) \vee (t \wedge s)$$

$$\underline{p \leftrightarrow (t \vee s)}$$

$$(p \wedge r) \rightarrow (q \wedge r)$$

Solution:

Idea: If there are such values of p, q, r, s that all assumptions are True(T), but conclusion is False(F) then the argument is Invalid. Otherwise it is Valid. Let's try to find such values.

(a.) When the conclusion $(q \to s)$ is False(F) is when $q = T$ and $s = F$.

If $q = T$, then the first assumption is T only for $p = T$, then $\neg p = T$, $\neg p \wedge q = F$, $\neg(\neg p \wedge q) = T$

If $p = T$, then the second asumption is T only for $r = F$, then $p \wedge r = F$, $\neg(p \wedge r) = T$.

If $r = F$ and $s = F$ then the third assumption is F.

Thus, all three assumptions can't be **True** at the same time when conclusion is F.

Therefore, the argument is **Valid**.

(b.) Partial Truth Table

p	q	r	s	t	$p \vee (\neg q)$	$(t \vee s) \to (p \vee r)$	$(\neg r) \vee (t \vee s)$	$p \leftrightarrow (t \vee s)$	$(p \wedge r) \to (q \wedge r)$
T	F	T	T	T	T	T	T	T	F

By referring to the partial truth table the concluion is **False** while all assumptions are **True**. Therefore, the argument is **Not Valid**

Problem 6: (Review Exercises Chapter 1 Question 15) Discuss the validity of the argument $p \wedge q$.

$$\underline{(\neg p) \wedge r}$$

Purple toads live on Mars.

Solution:

$(p \wedge q) \wedge ((\neg p) \wedge r)$

$\iff p \wedge q \wedge \neg p \wedge r \iff q \wedge r \wedge 0 \iff 0$

Since the assumption is always **False** because of the contradiction the implication is **True**.

Therefore, the argument is **Valid**.

Problem 7: (Review Exercises Chapter 1 Question 16) Determine the validity of each of the following arguments. If the argument is one of those listed in the text, name it.

(a.)

$$\begin{array}{c}\text{Either I wear a red tie or I wear blue socks.}\\ \underline{\text{Either I wear a green hat or I do not wear blue socks.}}\\ \text{Either I wear a red tie or I wear a green hat.}\end{array}$$

(b.)

$$\begin{array}{c}\text{If I like mathematics, then I will study.}\\ \text{Either I don't study or I pass mathematics.}\\ \underline{\text{If I don't pass mathematics, then I don't graduate}}.\\ \text{If I graduate, then I like mathematics.}\end{array}$$

Solution:

(a.) p: 1 wear a red tie.
q: 1 wear blue socks.
r: 1 wear a green hat.

The argument is

$$p \vee q$$

$$\underline{r \vee \neg q}$$

$$p \vee r$$

Valid by resolution since

$$p \vee q$$

$$r \vee \neg q$$

gives

$$p \vee r$$

.

(b.) p: 1 like mathematics.
q: 1 studies.
r: 1 passes mathematics.
s: 1 graduates.

The argument is

$$p \to q$$

$$\neg q \vee r \iff q \to r$$

$$\underline{\neg r \to \neg s \iff s \to r}$$

$$s \to p$$

Partial Truth Table

p	q	r	s	$p \to q$	$q \to r$	$s \to r$	$s \to p$
F	F	T	T	T	T	T	F

The partial truth table shows that the argument is **Not Valid** since the rguments are **True** while the conclusion is **False**.