MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ANSWERS FOR SAMPLE FINAL EXAM MATHEMATICS 2320 — APRIL, 2005

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1. Find converse, contrapositive and negation of statement:

If it snows tonight, I will stay at home.

Converse:*If I stay at home then it snows tonight.*

Contrapositive:*If I do not stay at home then it does not snow tonight.*

Negation: *It snows tonight and I will not stay at home.*

2. Give an example of an open statement and its existential quantification.

3. Construct thruth table for $(x \wedge y) \vee (\neg z \to x)$. Is it tautology or contradiction or none?

4. Is the following logical argument valid?

If you do every problem from this sample test then you will learn discrete math.

You learned discrete math.

Therefore you did every problem from this test.

Not valid.

5. Prove that $a^2 = b^2$ if and only if $a = b$ or $a = -b$.

the prove must consist from two parts:

Part 1: if $a = b$ or $a = -b$ then $a^2 = b^2$

Part2: if $a^2 = b^2$ then $a = b$ or $a = -b$

do simple algebra complete it!

6. Prove that n is even if and only if $7n + 4$ is even.

the prove must consist from two parts:

Part 1: if n is even then $7n + 4$ is even

Part2: if n is odd then $7n + 4$ is odd

7. Prove that $1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \cdots + n \cdot n! = (n+1)! - 1$ for all positive integer n.

By math induction:

1). n=1 then $1! = 2! - 1$

2) assume $1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \cdots + n \cdot n! = (n+1)! - 1$

and show (using the assumption) that

$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \cdots + n \cdot n! + (n+1)(n+1)! = (n+2)! - 1$.

3). Conclude that it works for all n.

8. Find the power set $\mathcal{P}(S)$ of set $S = \{\, a, \{a\}\,\}$. What is cardinality of the Cartesian product $S \times \mathcal{P}(S)$?

the power set $\mathcal{P}(S)$ has four elements.

the Cartesian product $S \times \mathcal{P}(S)$ contains 8 element.

9. Show that that the set of rational numbers is countable.

write the numbers a/b in a square and count diagonally (we did it in class and the book has it too)

10. Give an example of a finite non-empty partially ordered set without the maximum element.

{a,b}, {a,c}, {a},{b}, {c}

11. Let a binary relation be defined as:

two people are in the relation if their first names have the same number of letters.

Is this an equivalence relation or a partial order? If this is an equivalence relation, then what is the class of equivalence of *John*?

It is an equivalence relation.

All people whose name has 4 letters.

12. Is the function injective? surjective? bijective?

a) $f : Z \to Z_+$, $f(x) = x^4$. (here Z is the set of integers, Z_+ - non-negative integers).

Not injective, not surjevtive

b) $f : R \to R$, $f(x) = 2^x$. (here R is the set of real nambers).

injective, not surjective

13. Find sets A and B such that function $f : A \to B$,$f = (x-1)^{-1/2}$ is a bijection. Find its inverse.

$x > 1$, $y > 0$, $f^{-1}(y) = y^{-2} + 1$

14. A bowl contains 10 red balls and 10 blue balls. A women selects balls at random without looking at them.

a) how many balls must she select to be sure of having at least three balls of the same color?

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b) how many balls must she select to be sure of having at least three blue balls?

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15. English alphabet contains 21 consonants and five vowels. How many strings of six lowercase letters of the English alphabet contain

a) exactly 1 vowel ?

$5 \times 21^5 \times 6$

b) at least one vowel ?

$26^6 - 21^6$

16. What is the coefficient of x^9 in $(2-x)^{19}$?

$-C(19, 9)2^{10}$

17. How many ways are there to distribute hands of 5 cards to each of four players from the standard deck of 52 cards?

$52!/(5!\,5!\,5!\,5!\,32!)$

18. Solve $a_{n+2} = 6a_{n+1} - 9a_n$, $a_0 = 1$, $a_1 = 2$.

$a_n = (3)^n(1 - n/3)$