Math 3210 **Due Tue Nov 10** **Assignment #7**

1. Write the statement of the Cauchy-Goursat Theorem. Outline the proof.
2. Write the statement of the Cauchy Integral Formula. Outline the proof.
3. Write the statement of the Morera theorem. Outline the proof.
4. Evaluate the contour integral. Assume each contour to be positively oriented. Use Cauchy-Goursat Theorem or Cauchy Integral formula where applicable.
   (a) $\oint_{|z|=5} \frac{z^3}{z+4i}\, dz$
   (b) $\oint_{|z|=\sqrt{3}} \frac{z^2}{z+i+1}\, dz$
   (c) $\oint_{|z-\pi/4|=\pi/8} \cot z\, dz$
   (d) $\oint_{|z|=e} \frac{e^z}{z-3i}\, dz$
   (e) $\oint_{|z|=4} \frac{z^2}{(z+2i)(z+5)}\, dz$
   (f) $\oint_{|z|=2} (\cosh^2 z + \sinh^2 z)\, dz$
   (g) $\oint_{|z|=2} \frac{\cosh^2 z + \sinh^2 z}{z - \ln 2}\, dz$
   (h) $\oint_{|z-5|=2} \mathrm{Log}(z-2)\, dz$
   (i) $\oint_{|z|=3} \frac{2z+3}{(z+1)(z+2)}\, dz$
   (j) $\oint_{|z-i|=2} \frac{\sin z}{z^2+4}\, dz$
5. Show that if $f(z)$ is analytic within and on a simple closed contour C and z_0 is not on C, then

$$\int_C \frac{f'(z)}{z-z_0}\, dz = \int_C \frac{f(z)}{(z-z_0)^2}\, dz$$

6. Use Cauchy-Goursat Theorem to evaluate $\int_{-\infty}^{\infty} e^{-x^2} \cos(\pi x)\, dx$.
7. **Extra point problem.**
   Evaluate $\int_{|z|=2009} z^{-1} e^{2009z}\, dz$ and use this result to find $\int_0^{\pi} e^{2009\cos t} \cos(2009 \sin t) dt$.