Math 3210 **Due Tue Nov 3** **Assignment #6**

1. Draw the curve in the complex plane and classify it.

   You options are: simple (no self intersections); differentiable ($z'(t)$ is continuous); smooth (differentiable and $z'(t) \neq 0$); contour (union of finite number of smooth arcs); closed contour).

   (a) $z(t) = 1 + 3\cos t + i(2 + 3\sin t)$, $t \in [0, \pi/2]$

   (b) $z(t) = 2 + i\cos(2t)$, $t[0, \pi]$.

   (c) $z(t) = t + it^2$ for $t \in [0, 2]$ and $z(s) = (2 - s) + i(4 - 2s)$, $s \in [0, 2]$.

2. Let C be a contour given by $\{z(t), a \leq t \leq b\}$. Denote by $-C$ the contour which consists of the same points as C but has an opposite direction.

   (a) Show that $-C$ can be described as $\{z(-t), -b \leq t \leq -a\}$.

   (b) Prove that $\int_C f(z)dz = -\int_{-C} f(z)dz$ under the assumption that $f(z(t))$ is a piece-wise continuous function on $[a, b]$.

3. Let
$$z_1(t) = R_0 e^{-it}, \quad z_2(s) = \sqrt{R_0^2 - s^2} + is, \quad z_3(q) = q + i\sqrt{R_0^2 - q^2},$$
   where $R_0 > 0$ is a constant and t, s, q are real parameters. When the interval for t is given, $z_1(t)$ describes a curve in the complex plane. Give alternative descriptions of the same curve in terms of both $z_2(s)$ or $z_3(q)$ and find corresponding interval(s) for s and q.

   (a) $0 < t < \pi/6$, (b) $-\pi/2 < t < -\pi/4$, (c) $-3\pi/4 < t < \pi/2$.

4. Evaluate contour integral along curve $C = \{z(t), a \leq t \leq b\}$.

   (a) $\int_C (z^3 + 3z - 2)z^{-2}\, dz$, $z(t) = 3e^{2it}$, $0 \leq t \leq \pi$

   (b) $\int_C x|z|^{-2}\, dz$, $z(t) = \sqrt{3}e^{-it}$, $0 \leq t \leq 4\pi$

   (c) $\int_C \frac{1}{z} + \frac{1}{\bar{z}}\, dz$, $z(t) = \sqrt{3}e^{-it}$, $0 \leq t \leq 4\pi$

   (d) $\int xy + i(y^2 - x^2)/2dz$ with respect to piece-wise linear contour connecting points from $z = 2i$ to $z = 1$ and to $z = -1$.

5. Without evaluating the integral identify if the following estimate is True or False. **Explain.**

   (a) Let $C = \{2e^{2it}, 0 \leq t \leq \pi/4\}$. Check if $\left| \int_C \frac{z^2 + 2 + z}{z^5 + z + 1}\, dz \right| \leq \frac{4\pi}{15}$.

   (b) Let C be a line segment from $z = -1 + 2i$ to $z = 1$. Check if $\left| \int_C \frac{dz}{(z+1)^4} \right| \leq \frac{1}{\sqrt{2}}$.

6. **Extra Points Problem**

   Show that if $11z^{10} + 10iz^9 + 10iz - 11 = 0$ then $|z| = 1$.