Math 3210 **Due Thur Oct 22** **Assignment #5**

1. Evaluate and find *all values* of the expression
   (a) $(i-1)^{1+2i}$
   (b) $\frac{d}{dz}((-5)^z)$ at $z = i\pi$
   (c) P.V. $(-\sqrt{3}-3i)^3$

2. Given $\sin z = \dfrac{e^{iz}-e^{-iz}}{2i}$, $\cos z = \dfrac{e^{iz}+e^{-iz}}{2}$, $\sinh z = \dfrac{e^{z}-e^{-z}}{2}$, $\cosh z = \dfrac{e^{z}+e^{-z}}{2}$, $\tanh z = \dfrac{\sinh z}{\cosh z}$, $\operatorname{sech} z = \dfrac{1}{\cosh z}$ verify that
   (a) $\cos(2z) = \cos^2 z - \sin^2 z$
   (b) $|\sin z|^2 = \sin^2 x + \sinh^2 y$
   (c) $\sin(iz) = i\sinh z$
   (d) $(\tanh z)' = \operatorname{sech}^2 z$
   (e) $(\sec z)' = \sec z \tan z$

3. Consider equation $\tan w = z$.
   (a) Solve the equation and find w as a function of z
   (b) Find the derivative $\frac{d}{dz}w(z)$ in the simplest algebraic form.

4. Find derivative $w'(t)$ of the complex valued function w w.r.t. real variable t.
   (a) $w(t) = \sinh^2(\tan(3t) - it^3)$
   (b) $w(t) = (\sec t + i\operatorname{sech} t)^{n^2}$, $n = 1, 2, 3...$

5. (a) Evaluate integral $\int_0^\pi e^{(a+ib)t}dt$, where a, b are real numbers and t is real variable.
   (b) Use result from (a) to find $\int_0^\pi e^{at}\cos nt$, where a is real and n is a natural number.
   (c) Confirm your result in (b) using integration by parts.

6. Envaluate
   (a) $\displaystyle\int_1^\infty \frac{(t+i)^3}{t^{9/2}}\,dt$
   (b) $\displaystyle\int_0^\infty \left(\frac{1+i}{1+t}\right)^2 dt$
   (c) $\displaystyle\int_{-\infty}^\infty \frac{2}{1+t^2} + \frac{2it}{1+t^6}\,dt$

7. **Extra Points Problem** Find simple algebraic condition for real numbers k such that the principal value of the expression $(ik)^{ik}$ is real? Give few (approximate) numerical values of such numbers k.