Math 3210 **Due Thur Sept 29** **Assignment #2**

1. Find domain of the following functions and rewrite them in the form $u(x,y) + iv(x,y)$

   (a) $\dfrac{1}{z\bar{z}-5}$; (b) $\dfrac{1}{z+i}$; (c) $z + \dfrac{1}{z}$; (d) $\bar{z} + \bar{z}^3$.

2. Use de Moivre Identity to show that for any natural n

$$\cos(n\theta) = \mathrm{Re}\sum_{k=0}^{n} \cos^{n-k}\theta \sin^k\theta \frac{i^k n!}{(n-k)!k!}$$

   Apply the formula in the following

   (a) Find $\cos(8\theta)$ if $\cos\theta = 0.3$.

   (b) Let n be even. Show that

$$\cos(n\theta) = \sum_{m=0}^{n/2} \cos^{n-2m}\theta(1-\cos^2\theta)^m(-1)^m \frac{n!}{(n-2m)!(2m)!}$$

3. Rewrite in terms of z and $\bar{z}$ and simplify (a) $x^3 + y^3$; (b) $ix + \dfrac{x}{x^2+y^2} + y + \dfrac{iy}{x^2+y^2}$.

4. Let $f(z) = (z+i)^{-1}$. Find (a) $f(1/z)$; (b) $f(f(z))$; (c)$f(z+i)$.

5. Let $w = z^2 + i$.

   (a) Sketch domain in the z-plane which has rectangular image $-3 \le \mathrm{Re}w \le -2$, $2 \le \mathrm{Im}w \le 3$

   (b) Sketch domain in the z-plane which has image $\mathrm{Re}w > 0$ and $\mathrm{Im}w < 0$

6. Let $w = e^{\bar{z}}$, $z = x + iy$.

   (a) What is the image of line $x = 1$

   (b) What is the image of line $y = \pi/4$

   (c) Sketch the image of $\ln 2 \le x \le \ln 3$, $-\pi/3 < y < \pi/4$

   (d) Sketch domain in the z-plane which has image $\mathrm{Re}w > 0$ and $\mathrm{Im}w < 0$

7. **Extra Points Problem 1** Find the image of the set $Re(z) \ge 0$ under the transformation

$$w = \frac{z-1}{z+1}.$$

8. **Extra Points Problem 2** Show that for any integer n

$$\left(\frac{1+i\tan\theta}{1-i\tan\theta}\right)^n = \frac{1+i\tan(n\theta)}{1-i\tan(n\theta)}$$